Empower Teachers, Engage Students, Make Math Fun: Intervention That Does It All

Teacher Created Materials
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Two-Step Word Problems

Learning Objectives

Operations and Algebraic Thinking

- Solve two-step word problems using the four operations.

Mathematical Practices and Processes

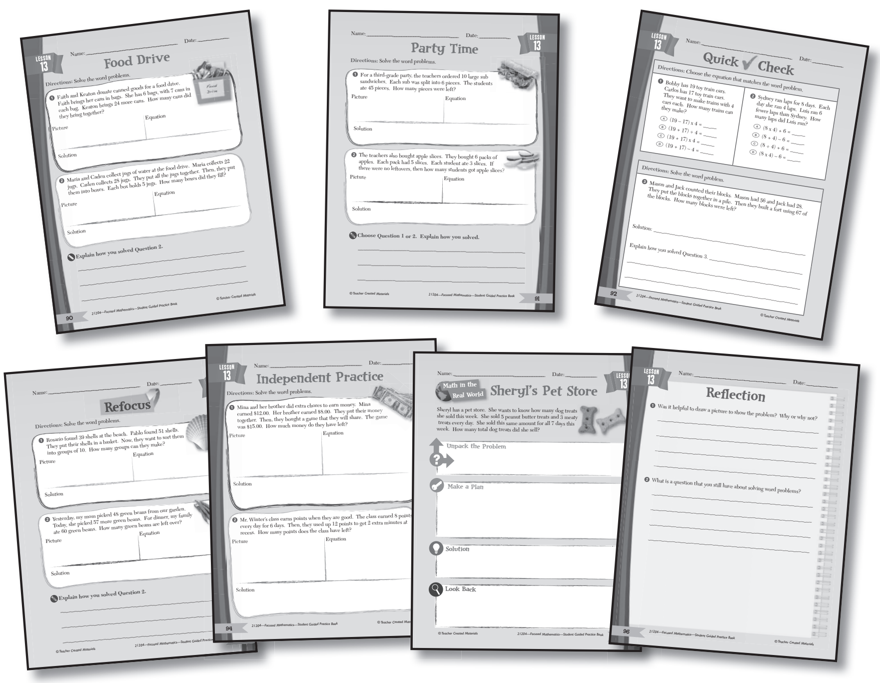
- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Model with mathematics.
- Use appropriate tools strategically.

Materials

- *Student Guided Practice Book* (pages 90–96)
- Math Fluency Game Sets
- Digital Math Fluency Games
- chart paper
- markers
- index cards

Progress Monitoring

The *Student Guided Practice Book* pages below can be used to formally and informally assess student understanding of the concepts.



Student Misconceptions

Students may struggle to make explicit connections between word problems and equations. Using a model for solving word problems in which students read to understand the problem, establish what they know, determine what they need to find out, and create and implement a plan for solving can help students experience success. Additionally, use of a problem-solving strategy, such as making a model or drawing a picture can help students relate what is happening in the problem to mathematical operations.

Two-Step Word Problems *(cont.)*

Warm-Up 10 min.

1. Prior to the lesson, create four large signs with chart paper. On each sign, write the symbol of a different operation (+, –, ×, ÷). Post the signs in different locations.
2. Say, "Today, we are going to play 'Find the Sign.' To play, you will listen to a word problem. Then, you will decide which operation should be used to solve the problem. You will choose addition, subtraction, multiplication, or division." Point out the four signs. Say, "Once you have decided on the operation, walk to that sign."
3. Practice a round. Read aloud the following: "Danny has 38 marbles. He gives 15 to his friend. How many marbles does Danny have left?" Allow students time to decide on, and walk to, their sign. Have students discuss the reasoning for choosing their sign, ultimately concluding that the correct operation is subtraction.
4. Play "Find the Sign." You can use these word problems:
    - Maria has 4 bags of apples. Each bag has 5 apples. How many apples does she have altogether? *(multiplication)*
    - Gus had 86 trading cards. He got 15 more cards for his birthday. How many cards does he have now? *(addition)*
    - I have 20 hair clips. I want to sort them into groups of 2. How many groups will I have? *(division)*
    - Tomas had 50 balloons. He sold 38 of them at the carnival. How many balloons are left over? *(subtraction)*

Language and Vocabulary 10 min.

1. Prior to the lesson, write the following terms on separate index cards.

    equation **unknown** **multiply** **divide** **subtract** **add**

2. As a class, examine how you could sort and group the cards. Give students a category. Ask, "Which terms would belong in that category? How are the terms related?" Possible ideas for categories (and the corresponding terms) are listed below.
    - Operations: *add, subtract, multiply, divide*
    - Solve: *equation, unknown*

Two-Step Word Problems *(cont.)*

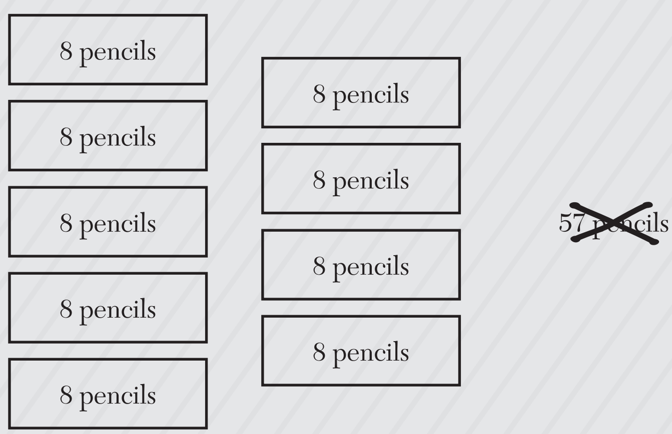
Whole-Group Lesson 40 min.

Focus

1. The following lesson will address this focus question:
   How can you use different strategies to solve word problems?
2. You may wish to write the focus question on the board and read it aloud to students. Explain that you will revisit the focus question at the end of the lesson.

I Do

1. Say, "Today, we will solve word problems." Write the following problem on the board. Read the problem as you write: *The math team bought 9 boxes of pencils. Each box had 8 pencils. During the school year, they used 57 pencils. How many pencils are left?*
2. Say, "First, let's unpack the problem and determine what we know. What does the problem tell us?" Circle key phrases as students share. They should indicate that there were nine boxes, with eight pencils in each box. Then, 57 pencils were used. Ask, "What do we need to find out?" When students successfully indicate that we need to know how many pencils are left, underline this portion of the problem.
3. Say, "Now, let's make a plan to solve. First, we can draw a picture to visualize the problem. Then, we will write an equation." Draw a picture to model the problem, for example:



4. Say, "Now, let's use our picture to write an equation. We need to find out how many pencils are left. But, how can we find how many pencils we started with?" Circle the nine rectangles you drew. Students should identify that we need to determine how many pencils are in nine boxes of eight.

Two-Step Word Problems *(cont.)*

Whole-Group Lesson *(cont.)*

I Do (cont.)

5. Ask, "What operation will help me figure out how many are in nine boxes (or groups) of eight?" When students can identify multiplication, write 9×8 on the board.
6. Ask, "Is there any other information we need to add to our equation to figure out how many pencils are left?" Students should identify the 57 pencils that were used. Ask students to determine the correct operation (*subtraction*) and add – 57 to the equation. Finally, add parentheses around the phrase 9×8, explaining that you need to solve this part of the equation first. The equation should be: $(9 \times 8) - 57 =$ _____.
7. Say, "To find the solution, we will solve the equation in order. What should we do first?" Students should indicate to solve what is in the parentheses. Ask students to identify the product (72) and rewrite the equation as $72 - 57$. Model how to find the difference using a method that students should be familiar with (*difference = 15*).
8. Say, "We solved our equation and found a solution of 15. Let's look back at our problem to see what 15 means." Encourage students to reread the problem. Students should identify that 15 describes the number of pencils that the math team has left. Write this as a complete sentence (*There are 15 pencils left*).

We Do

1. Refer students to the Food Drive activity sheet (*Student Guided Practice Book*, page 90). Say, "Let's solve more word problems together." First, read the problem: *Faith and Keaton donate canned goods for a food drive. Faith brings her cans in bags. She has 6 bags, with 7 cans in each bag. Keaton brings 24 more cans. How many cans did they bring together?*
2. Say, "First let's unpack the problem and look at what we know. Circle words or phrases that tell us key information." Allow students to circle key words, and then share their ideas. Students should circle phrases such as: *donate canned goods*; 6 *bags, with 7 cans in each bag;* and *Keaton brings 24 more cans*. Ask, "What do we need to find out?" When students successfully indicate that we need to find out how many cans were donated altogether, have them underline this portion of the problem.

Two-Step Word Problems *(cont.)*

Whole-Group Lesson *(cont.)*

We Do (cont.)

3. Tell students that they will draw a picture to visualize the problem. Allow them to discuss ideas with a partner, and then draw. Ask students to share their drawings. They should create some representation of the six bags, with seven cans in each, for example six circles with seven dots in each circle. To show the additional 24 cans, they may have written the numeral or drawn 24 dots or circles.
4. Say, "Now, you will use your picture to write an equation. We said we needed to find out the total number of cans that were donated. What operation can we use to show this?" When students suggest addition, ask what needs to be added together. Help students recognize that Faith's and Keaton's cans need to be added, and then have students write an addition symbol in the *Equation* box on their activity sheet.
5. Ask, "How many cans did Faith bring?" Encourage students to look back at their drawing to determine that she brought six bags of seven cans. Ask them to think about how they could represent this mathematically. If needed, help them record 6×7 to the left of the addition symbol. Explain that we can use multiplication because the bags are like equal groups. Ask, "How many cans did Keaton bring?" When students identify 24 cans, have them write this on the other side of the addition symbol. Students should also add parentheses around the multiplication portion of the equation, to show that this should be solved first. The equation should read: $(6 \times 7) + 24$.
6. Have students solve the equation, and then share their solution. Remind them to first solve what is in the parentheses, and then add. Students should arrive at a solution of 66. Encourage them to look back at the problem and write the solution as a complete sentence answer to the question (e.g., *Together, Faith and Keaton brought 66 cans*).
7. Repeat the process with Question 2 on the activity sheet. Finally, have students explain how they solved Question 2. To assist them, provide the following sentence frames:
    - *My picture shows _____.*
    - *I used the equation _____ to solve the problem.*
    - *Maria and Caden filled _____ boxes.*

Language Support

Allow students to complete the Party Time activity sheet with a student partner. The partner should assist in correctly decoding directions and assisting in comprehension of the word problem.

Two-Step Word Problems *(cont.)*

Whole-Group Lesson *(cont.)*

You Do

1. Tell students they will now work on solving two-step word problems on their own on the Party Time activity sheet (*Student Guided Practice Book*, page 91). Provide the sentence frames from Step 7 of the We Do section to help students explain their reasoning.
2. Have students share their solutions, including the equations that they wrote to solve the word problems. If students have difficulty explaining their reasoning, remind them to use the sentence frames and vocabulary terms.

Closing the Whole-Group Lesson

Revisit the focus question for the lesson: *How can you use different strategies to solve word problems?* Ask students to identify the different strategies used in the lesson to solve word problems (drawing a picture and writing an equation). Have students identify something they found challenging about solving two-step word problems. Ask them to explain how the problem-solving strategies helped them to work through those challenges.

Progress Monitoring 5 min.

1. Have students complete the Quick Check activity sheet (*Student Guided Practice Book,* page 92) to gauge student progress toward mastery of the Learning Objectives. Provide students with unlined paper to show their work on the selected response questions.
2. Based on the results of the Quick Check activity sheet and your observations during the lesson, identify students who may benefit from additional instruction in the Learning Objectives. These students will be placed into a small group for reteaching. See instructions on the following page.

Two-Step Word Problems *(cont.)*

Differentiated Instruction 20 min.

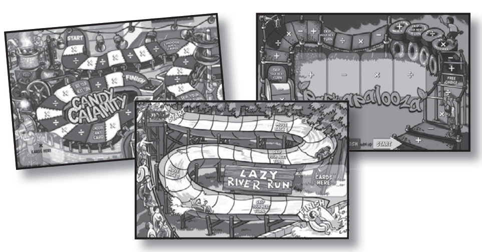
Gather students for reteaching. The remaining students will complete the Independent Practice activity sheet (*Student Guided Practice Book*, page 94) to reinforce their learning and then play the Math Fluency Games.

Refocus PPT

Revisit the focus question for the lesson: *How can you use different strategies to solve word problems?* Help students focus on understanding and representing key information. Model how to identify key information for Question 1 on the Refocus activity sheet (*Student Guided Practice Book,* page 93): *Rosario found 39 shells at the beach. Pablo found 51 shells. They put their shells in a basket. Now, they want to sort them into groups of 10. How many groups can they make*? Sentence by sentence, model how to circle key information, especially numbers. Then, look at this information together and discuss what the key information means. What does this number represent? What do I need to find out to solve the problem?

Together with students, draw a picture to represent the key information. For example, draw a circle to represent the basket with *39 shells* and *51 shells* written inside. Then draw 10 circles with the phrase *10 groups* below. Look at each piece of key information to generate the equation to solve the problem: $(39 + 51) \div 10 =$ _____. Model how to solve the equation one step at a time. Finally, have students solve Question 2 with a partner, providing support as needed. Students will also explain how they solved Question 2.

Math Fluency Games



Math Fluency Game Sets

Digital Math Fluency Games

Extend Learning

Have students complete the Lesson 13 Extend Learning Task (filename: extendtask13.pdf). Students will create their own two-step problems to match the equation on the activity sheet. Be sure that students solve their word problem.

Two-Step Word Problems *(cont.)*

Math in the Real World 30 min.

1. Refer students to the Math in the Real World: Sheryl's Pet Store task (*Student Guided Practice Book*, page 95). Have a student read the task aloud. Tell students to explain or summarize the task to their partner. Have a few students share their summaries.
2. Ask students to think about what information they will need to solve the task and what the task is asking them to do. Then, have them share with a partner. Ask a few students to share out. Students should identify that we know how many of each treat Sheryl sold per day. We also know how many days she sold this number of treats. We need to find how many treats she sold in all. Have students work in groups of two or three to complete the task.
3. As students are working, circulate and ask focusing, assessing, and advancing questions:
   - *How can you draw a picture to show the key information in the problem?*
   - *How can you find the total number of treats sold in one day? How can you find the number of treats sold in seven days?*
   - *How can you represent this problem with an equation?*

Sentence Frames for Explaining Reasoning

- *To solve this problem, I need to find out _____.*
- *I wrote the equation _____ to solve the problem.*
- *Sheryl sold _____ dog treats at her store.*

4. Observe how students are solving the task, and choose a few groups who solved the task in different ways to share their solutions and reasoning. Try to have the solutions move from concrete representations to more abstract representations. For example, have one group share their visual representation and then have another group share their equation. Discuss whether the two are a match. Make sure students explain their reasoning as they share solutions.
5. As groups are sharing their solution paths, reasoning, and strategies, ask questions:
   - *Did you write the same equation? If not, what equation did you write?*
   - *Do you agree or disagree with the solution path and reasoning? Why?*
   - *Which solution path makes the most sense to you? Why?*

Lesson Reflection 5 min.

Have students summarize their perspectives on drawing pictures to visualize problems, and provide feedback on any questions they still have about the content on the Reflection activity sheet (*Student Guided Practice Book*, page 96).

Name:_________________________ Date:_______________

Performance Task 1:
Pancake Breakfast

Part A

There will be a pancake breakfast at Park View School. Eight tables are available. Each table can seat 6 people. How many people can be seated at the breakfast? How do you know this?

Solution: ______________________________

Explain your solution.

____

____

____

____

____

Name:_______________________________________ Date:____________________

Performance Task 1: Pancake Breakfast *(cont.)*

Part B

There are 4 more tables available for the breakfast. How many people can be seated at the breakfast now? How do you know this?

Solution: ____

Explain your solution.

____

____

____

____

____

Name:____________________________________ **Date:**____________________

Performance Task 1: Pancake Breakfast *(cont.)*

Part C

Look back at your solution to Part B. Now read Part C.

Amber says, "72 people can be seated at the breakfast because $6 \times 12 = 72$."
Juan says, "72 people can be seated because $72 \div 6 = 12$ tables."
Who is correct?

Solution: ____

Explain your solution.

____

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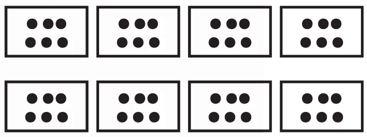
____

____

Performance Task Answer Key

Performance Task 1: Pancake Breakfast

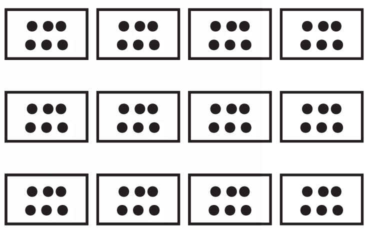
Part A (page 43)



$8 \times 6 = 48$

Solution: 48 people can be seated.
Answers will vary; should include multiplying the number of tables by the number of people at each table: $8 \times 6 = 48$.

Part B (page 44)



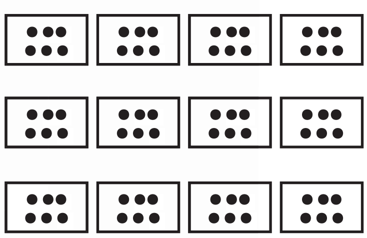
$8 \times 6 = 48$

$4 \times 6 = 24$

$48 + 24 = 72$

Solution: 72 people can now be seated.
Answers will vary; may include multiplying the new number of tables by the number of people at each table, then adding that to the previous answer:
$4 \times 6 = 24$
$24 + 48 = 72$
Alternatively, answer may include adding the new number of tables to the previous number of tables and then multiplying that number by the number of people at each table: $8 + 4 = 12$
$12 \times 6 = 72$

Part C (page 45)



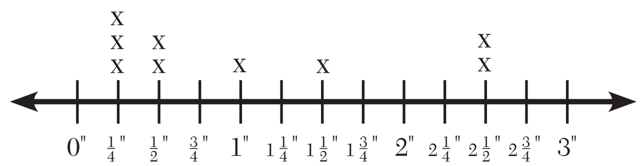
$6 \times 12 = 72$

$72 \div 6 = 12$

Solution: Both students are correct.
Answers will vary; may include that both students are correct because the picture for both equations is the same (if pictures are used) or because $6 \times 12 = 72$ and $72 \div 6 = 12$ are related facts.

Performance Task 2: Charts and Graphs

Part A (page 46)



Part B (page 47)

1. May
2. There were 30 more oranges. Answers will vary; may include that each section is 5 oranges, and the difference between May and March is 6 sections. $5 \times 6 = 30$.
3. There were 5 fewer oranges. Answers will vary; may include that each section is 5 oranges, and the difference between April and June is only one section.

Part C (page 48)

1. Cats are the most popular pet.
2. 2 more people own fish and hamsters than dogs. Answers will vary; should include that 4 people own fish and 5 people own hamsters, which is a total of 9 owners. 7 people own dogs. $9 - 7 = 2$.
3. 18 students own dogs, cats, and guinea pigs. $7 + 9 + 2 = 18$
4. The scale is 2.

Performance Task 1 Rubric

Pancake Breakfast				
	Part A	**Part B**	**Part C**	
Math Content Criteria	The student correctly identifies 48 as the number of possible attendees. The student accurately makes a drawing or equation showing that 8×6 is used and explains his or her thinking.	An equation or model is used to arrive at the answer (72 people). The student accurately explains his/her thinking in connecting the two steps.	The student correctly states that both answers are correct. A model (array, equation, or number line) is used to justify the answer.	
Content Score	_____ / 2 pts	_____ / 2 pts	_____ / 2 pts	
Math Process Criteria	Make sense of problems and persevere in solving them. • Work indicates some type of planning was involved: simpler problems, models, or equations showing the steps. Model with mathematics. • The student accurately builds arrays, uses a number line, or writes equations/expressions.	Make sense of problems and persevere in solving them. • Work indicates some type of planning was involved: simpler problems, models, or equations showing the two steps. Model with mathematics. • The student accurately builds arrays, uses a number line, or writes equations/expressions.	Make sense of problems and persevere in solving them. • Work indicates some type of planning was involved: simpler problems, models, or equations showing the steps. Construct viable arguments and critique the reasoning of others. • There is a logical explanation indicating that both answers are correct. Model with mathematics. • The student accurately builds arrays, uses a number line, or writes equations/expressions.	
Process Score	_____ / 2 pts	_____ / 2 pts	_____ / 3 pts	
TOTAL	**Part A:** _____ / 4 pts	**Part B:** _____ / 4 pts	**Part C:** _____ / 5 pts	**Total Points:** _____ / 13 pts

LESSON 13

Name: ______________________ Date: __________

Food Drive

Directions: Solve the word problems.

1. Faith and Keaton donate canned goods for a food drive. Faith brings her cans in bags. She has 6 bags, with 7 cans in each bag. Keaton brings 24 more cans. How many cans did they bring together?

Picture

Equation

Solution

2. Maria and Caden collect jugs of water at the food drive. Maria collects 22 jugs. Caden collects 28 jugs. They put all the jugs together. Then, they put them into boxes. Each box holds 5 jugs. How many boxes did they fill?

Picture

Equation

Solution

Explain how you solved Question 2.

____

____

____

Name:______________________________ Date:____________

LESSON 13

Party Time

Directions: Solve the word problems.

1. For a third-grade party, the teachers ordered 10 large sub sandwiches. Each sub was split into 6 pieces. The students ate 45 pieces. How many pieces were left?

Picture	Equation

Solution

2. The teachers also bought apple slices. They bought 6 packs of apples. Each pack had 5 slices. Each student ate 3 slices. If there were no leftovers, then how many students got apple slices?

Picture	Equation

Solution

Choose Question 1 or 2. Explain how you solved it.

____

____

____

Name: ______________________________ Date: ____________

Quick ✓ Check

Directions: Choose the equation that matches the word problem.

1. Bobby has 19 toy train cars. Carlos has 17 toy train cars. They want to make trains with 4 cars each. How many trains can they make?

   Ⓐ (19 – 17) x 4 = _____

   Ⓑ (19 + 17) ÷ 4 = _____

   Ⓒ (19 + 17) x 4 = _____

   Ⓓ (19 + 17) – 4 = _____

2. Sydney ran laps for 8 days. Each day she ran 4 laps. Luis ran 6 fewer laps than Sydney. How many laps did Luis run?

   Ⓐ (8 × 4) + 6 = _____

   Ⓑ (8 + 4) – 6 = _____

   Ⓒ (8 ÷ 4) + 6 = _____

   Ⓓ (8 × 4) – 6 = _____

Directions: Solve the word problem.

3. Mason and Jack counted their blocks. Mason had 56 and Jack had 28. They put the blocks together in a pile. Then they built a fort using 67 of the blocks. How many blocks were left?

Solution: ____

Explain how you solved Question 3. ____

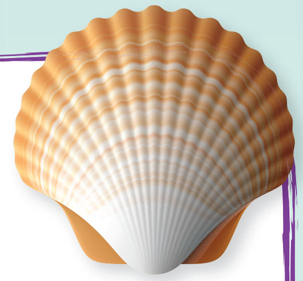
____

____

Name:______________________________ Date:__________

Refocus

Directions: Solve the word problems.



1. Rosario found 39 shells at the beach. Pablo found 51 shells. They put their shells in a basket. Now, they want to sort them into groups of 10. How many groups can they make?

Picture

Equation

Solution

2. Yesterday, my mom picked 48 green beans from our garden. Today, she picked 57 more green beans. For dinner, my family ate 60 green beans. How many green beans are left over?

Picture

Equation

Solution

Explain how you solved Question 2.

____

____

____

LESSON 13

Name:______________________ Date:__________

Independent Practice

Directions: Solve the word problems.

1 Mina and her brother did extra chores to earn money. Mina earned $12.00. Her brother earned $8.00. They put their money together. Then, they bought a game that they will share. The game was $15.00. How much money do they have left?

Picture

Equation

Solution

2 Mr. Winter's class earns points when they are good. The class earned 8 points every day for 6 days. Then, they used up 12 points to get 2 extra minutes at recess. How many points does the class have left?

Picture

Equation

Solution

Name:______________________________ Date:__________

LESSON 13

Math in the Real World

Sheryl's Pet Store

Sheryl has a pet store. She wants to know how many dog treats she sold this week. She sold 5 peanut butter treats and 3 meaty treats every day. She sold this same amount for all 7 days this week. How many total dog treats did she sell?

Unpack the Problem

Make a Plan

Solution

Look Back and Explain

LESSON 13

Name: ______________________ Date: ____________

Reflection

1. Was it helpful to draw a picture to show the problem? Why or why not?

____

____

____

____

2. What is a question that you still have about solving word problems?

____

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Research-Based Curriculum

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Research on Mathematics Intervention

In 21st century teaching and learning, the necessity of mathematical literacy and mathematical education reform have become more important than ever. In a society that has become so technically oriented, "innumeracy" has replaced illiteracy as our principal educational gap. This is the age of science, technology, and mathematics. To have a mathematically literate society, the population needs to have understanding of, and proficiency with, mathematics concepts and procedures, as well as the ability to apply that knowledge, use it to develop models, and apply those models to new situations.

The goal of mathematics education is to provide all students with the ability to use mathematics to improve their own lives, to help them become aware of their responsibilities as citizens, and to help them prepare for their futures.

In order to accomplish these goals, state departments of education, school districts, and teachers must set high expectations for all students, and mathematics education needs to be a priority at all levels. *PISA 2012 Assessment and Analytical Framework: Mathematics, Reading, Science, Problem Solving and Financial Literacy describes* the expectations students are to meet and the experiences they need to have to achieve them. "This conception of mathematical literacy supports the importance of students developing a strong understanding of concepts of pure mathematics and the benefits of being engaged in explorations in the abstract world of mathematics. The construct of mathematical literacy, as defined for PISA, strongly emphasizes the need to develop students' capacity to use mathematics in context, and it is important that they have rich experiences in their mathematics classrooms to accomplish this" (PISA 2013, 25).

Mathematics education must begin at a very early age so that students develop the foundational understanding and skills necessary to achieve in mathematics. More instructional time should be dedicated to mathematics instruction, and the curriculum should focus on a depth versus breadth approach so that students have sufficient opportunities to achieve and master the content. Cameron (et al.) emphasized the importance of concepts being taught thoroughly and solidly. Students should be solving problems that require higher-level thinking and address grade-level topics (2011). Teachers need research-based curriculum solutions to provide strong instructional support that will develop mathematical proficiency among all students.

Researchers have focused their efforts in recent years on identifying essential elements of effective mathematical interventions. These include explicit, systematic, problem-based instruction in:

1. developing proficiency in number sense with whole and rational numbers
2. building accuracy and fluency in arithmetic combinations
3. building conceptual knowledge and procedural understanding
4. problem solving

(Gersten et al. 2009)

The Need for Intervention

Students come to the classroom with different learning styles, various levels of mathematical proficiency, language barriers, communication issues, and diverse backgrounds. Teachers must understand the development of mathematics, considering the progression of concepts, strategies, and models that can become powerful forms of representation and tools to think with (Fosnot and Hudson 2010). Student attitudes and personalities also affect learning. Many students suffer from math anxiety, an anxiety often supported by parents who reflect their own negative perceptions of mathematical abilities onto their children. Some students find math boring or unnecessary. And some students may do poorly simply because they have low self-esteem when it comes to math. Other issues stem from the way in which students access mathematical content. Some students struggle to visualize or develop understanding of abstract concepts. Other students struggle to master mathematical procedures because they do not understand the concept or the rationale for the steps of the procedure. Additionally, many students do not possess strategies for attacking an unfamiliar word problem. Whatever the obstacle, it is essential that our education systems try to meet the mathematical needs of all students before they fail. This is why intervention is critical.

There is a need on a national level for all students, especially students with math difficulties, to deepen their understanding of and proficiencies in mathematical concepts and processes. In 2000, the National Council of Teachers of Mathematics (NCTM) released the "Principles and Standards for School Mathematics: An Overview," which deepened the understanding that mathematics is a combination of content and process, encouraging the expectation of standards-based teaching (NCTM 2000). Following its release, a project sponsored by the National Science Foundation and the U.S. Department of Education published "Adding It Up: Helping Children Learn Mathematics" (National Research Council 2001). This publication introduced the five strands of mathematical proficiency. The intent of the report was to ensure that students become proficient in math content and processes. This laid the groundwork for agencies such as the National Governors Association Center for Best Practices and the Council of Chief State School Officers to develop mathematical content and practice/process standards that focus on the conceptual and procedural understanding children must have to develop mathematical proficiency (2014). The standards are designed as progressions, each level building upon the next. The documents are interconnected works that describe the expertise that all mathematics educators should develop in their students to build their proficiencies in mathematical understanding, reasoning, and application. Ultimately, these documents, and the state standards that have evolved from them, are designed to close the education gap and provide all students equal opportunity to achieve mathematical literacy.

"*Mathematical literacy* is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena" (PISA 2013, 25).

Response to Intervention in Mathematics

Response to Intervention (RTI) models create an integrated education system that builds a level of support for all students to be successful. In "Why Adopt an RTI Model?" David P. Prasse, Ph.D., reveals the importance and need for RTI. Since Public Law 94-142, the Education for All Handicapped Children's Act (EHA), was signed in 1975, educators focused on the identification and placement of special education students (2014). Student outcomes, achievement, and delivery of effective interventions were not measured. A system was needed to address academically struggling students in the general education setting.

After 25 years, educators began to implement RTI in special education settings to measure student outcomes, recognizing the need for identification before failure and for integrating general education and special education. According to Prasse, RTI is designed to provide all students with an education that utilizes effective intervention programs and that is based on frequent progress monitoring (2014).

RTI is generally a multitiered instructional model, including universal screening and progress monitoring assessments, that increases the intensity of the intervention based on the nature and severity of a student's difficulties. Specific definitions of the tiers differ from state to state, but the following are general descriptions based on the article "Tiered Instruction and Intervention in a Response-to-Intervention Model" by Edward S. Shapiro, Ph.D., RTI Action Network (2014).

Tier 1 is considered Core Instructional Intervention. It starts with high-quality, standards-based instruction in the general education classroom and the use of research-based curriculum. To meet the diverse needs of students during core instruction, teachers differentiate by content, process, and product. Universal screening and progress monitoring is used in this setting to differentiate instruction and monitor students' levels of mastery of the learning objectives and state standards.

Research to Practice

Tier 1

Teachers can use *Focused Mathematics Intervention* as a supplement to the core curriculum. The focused lessons provide explicit instruction in key skills that students need, with options for differentiating instruction, including extending learning for students who have shown mastery of the skill.

Tier 2 is designed to meet the needs of those students who perform below expected grade-level benchmarks or standards after Tier 1 instruction. These students are at levels that indicate some risk for failure. Tier 2 students' needs are identified through the assessment process, and these children receive supplemental intervention in addition to Tier 1 instruction. Instruction is delivered in a small-group setting to address the identified needs, and progress is monitored through program assessments.

> **Research to Practice**
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> **Tier 2**
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> After administering the Pretest, teachers may work with a small group of students who require focused instruction on a particular area of need. Teachers can monitor student progress using the Quick Checks and informal assessment opportunities, such as student responses during activities, embedded in each lesson.

Tier 3 provides intervention in a setting with three to five students or one on one. It is designed to meet the needs of students who have demonstrated that they are far below grade-level standards and at high risk of failure. In many models, Tier 3 is considered special education. Students in Tier 2 and Tier 3 may participate in the same intervention program, but students in Tier 3 may attend more frequently and/or for longer periods of time. Student progress is often monitored at a higher level of frequency than in Tier 2 (Shapiro 2014).

> **Research to Practice**
>
> **Tier 3**
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> Teachers can use the Pretest to determine a student's specific areas of need, and then deliver targeted lessons to address those needs in a one-on-one setting. At Tier 3, providing extra time and allowing opportunities to practice and review are appropriate. If a student needs to focus on a specific skill, such as working with a specific model, focus on this section of the lesson. The Refocus portion of the lesson can be effective for such review.

Components of Effective Mathematics Interventions

In 2008, after a review of the research in effective mathematics instruction, the National Mathematics Advisory Panel (NMAP) summarized the six main steps that need to be taken to improve math learning in U.S. schools:

1. Pre-kindergarten through grade 8 math curricula should be streamlined to emphasize a narrower set of the most critical topics in math.
2. There must be better knowledge of how students learn mathematical concepts and the benefits of intervention, conceptual understanding, fluency, automaticity, and math skill development.
3. Teachers must have strong math skills in order to teach math.
4. Math instruction should be a combination of student- and teacher-focused instruction.
5. Assessments should be strengthened to include the emphasis of the most critical math knowledge and skills.
6. More rigorous math research designed to improve best teaching practices is needed.

Research to Practice

Components of Effective Mathematics Intervention

Focused Mathematics Intervention supports the NMAP six steps to improving math learning in the following ways:

- Rigorous and explicit lesson design provides research-based practices in math instruction through the gradual release of responsibility model.
- Best practices and models for teachers are integrated through descriptions of student misconceptions and teacher background, as well as a teacher glossary.
- Formative and summative assessments help instructors diagnose, decide, and deliver rigorous instruction.

Build Mathematical Proficiencies

The majority of students' math difficulties are with number sense, accuracy in arithmetic combinations, and problem solving (Hanich et al. 2001). *Focused Mathematics Intervention* supports struggling math learners using systematic, explicit instruction throughout all lessons. Mathematics concepts are taught through a developmental lens, allowing strategies and mathematical models to be used as tools for instruction, student understanding, and reasoning.

Research to Practice

Build Mathematical Proficiencies

Focused Mathematics Intervention lessons provide the following:

- opportunities for students to engage in a variety of rigorous mathematical problem types
- focused instruction on key foundational skills to focus on both whole and rational numbers, arithmetic combinations, geometry, measurement, data, and probability
- active application of mathematical word/story problem skills
- multiple opportunities to explain mathematical reasoning
- reinforcement of key math fluency skills through cooperative math fluency games

Direct, Sequential, and Gradually Released Instruction

It is essential to create an engaging learning environment in which students' mathematical understanding grows through systematic, explicit modeling, with multiple opportunities for guided and independent problem solving.

A "structure for instruction that works" is the gradual release of responsibility (Fisher and Frey 2008). The teacher guides students through the following sequence to build mastery of the standard and student ownership of learning.

• Focus Lesson (I Do)
The teacher explicitly guides conceptual development or gives explicit instruction of the skill by activating prior knowledge, conducting Think Alouds, establishing investigative questions, and modeling mathematical examples and tasks.

- **Guided Instruction (We Do)**
The teacher and students further develop understanding of the concept or build procedural proficiency by asking questions, participating in mathematical discourse, formulating/testing hypotheses, and working mathematical tasks together.

- **Collaborative (You Do)**
Students collaboratively build mastery of the concept or skill. While working mathematical tasks, students explore mathematical conjectures, ask and respond to questions, clarify understanding, provide feedback, and reflect on their work.

- **Independent (You Do)**
Students independently solidify their mastery of the concept or skill. While working mathematical tasks, students progress through the problem-solving process, ask themselves questions, conduct self-talk, and explain their mathematical understanding in writing.

Research to Practice

Direct, Sequential, and Gradually Released Instruction

Every *Focused Mathematics Intervention* lesson utilizes a gradual release of responsibility model (I Do, We Do, You Do):

- teaches students how to be active math learners through explicit instruction of mathematics strategies
- provides students with support to ensure the successful transfer of key mathematical concepts and procedures from guided practice to independent application

High-Yield Strategies for Increasing Student Achievement

Marzano, Pickering, and Pollock (2001) have identified nine high-yield strategies for improving instruction and student achievement: identifying similarities and differences; summarizing and note-taking; reinforcing effort and providing recognition; homework and practice; nonlinguistic representations; cooperative learning; setting objectives and providing feedback; generating and testing hypotheses and questions; and advanced organizers. These nine strategies have the greatest measurable positive effect on all student achievement, regardless of grade level or subject matter. The work of these researchers was incorporated into the development of *Focused Mathematics Intervention*.

Research to Practice

High-Yield Strategies for Increasing Student Achievement

The *Focused Mathematics Intervention* series incorporates the nine high-yield strategies in the following ways:

- Each **lesson objective is clearly set** on the lesson overview page and reinforced in the focus question(s).
- The teacher uses data from checks for understanding and formative and summative assessment to **provide feedback, reinforce effort, and provide recognition**.
- The constructivist approach of each lesson allows students to explore mathematical concepts by **asking their own questions, formulating hypotheses, and testing those hypotheses** through construction of mathematical representations, modeling, and problem solving.
- The gradual release of responsibility lesson design includes **opportunities for practice** in each phase of instruction, as well as in Differentiated Instruction, Math Fluency Games, and Math in the Real World concept tasks.
- The Warm-Up and focus question **provide questions and cues** prior to presenting new content to activate and link to prior knowledge.
- Students may use a **math journal to take notes** during the Warm-Up and Language and Vocabulary. Students may also benefit from **writing responses, summarizing mathematical thinking, and recording model problems** during the Whole-Group lesson.
- Students participate in **cooperative learning** during the We Do, Math Fluency Games, and Math in the Real World concept tasks.
- The Warm-Up, Language and Vocabulary, and *Student Guided Practice Book* activities allow students to explore mathematical relationships and **identify similarities and differences among math concepts**.
- Through the use of math manipulatives, multiple representations, and modeling, students **utilize nonlinguistic representations** to develop conceptual understanding, transition to abstract models, and problem solve.

Using Technology to Improve Mathematical Learning

Within the last decade there has been a shift in the ways students think and process information. Unlike their predecessors, students in today's classroom have been deemed *digital natives*. They were born into a digital world and have developed thinking patterns that are different from those of previous generations (Pressley 2001). This pedagogical shift has been recognized in the flux of digital technologies offered in today's classroom. Moreover, according to focal points of 21st century learning, technology should be used widely and responsibly in the classroom—with the goal of enriching students' learning of language.

Educators are challenged with preparing all students for a more technologically advanced world (Harwood and Asal 2007).

Extensive research has been conducted over the years to determine how effective technology is in improving student performance. The following positive effects have been observed:

- increased student achievement
- improved higher-order thinking skills and problem-solving abilities
- enhanced student motivation and engagement
- improved ability to work collaboratively

Research to Practice

Using Technology to Improve Mathematical Learning

Each level of *Focused Mathematics Intervention* features a variety of digital resources that allow teachers to weave technology into mathematics instruction.

Digital Resources:

- Refocus Mini Lessons (PowerPoint®/ PDFs)
- Digital Math Fluency Games
- PDFs of all Math Fluency Game Sets
- Teacher resources in multiple file formats
- PDFs of all student pages and assessments
- User Guide

Electronic Assessments: Electronic versions of the Pretest, Posttest, and reporting tools are included on the Digital Resources USB Device.

Using Games to Motivate Struggling Math Learners

Games are a proven source of motivation. They are a fun way for students to develop, maintain, and reinforce mastery of essential mathematical concepts and processes. Games eliminate the tedium of most mathematics skill drills. The article "Gamification in Education: What, How, Why Bother?" by Lee and Hammer (2011) discusses the benefits and learning potential of using games in the classroom. Citing a variety of research (Locke 1991; Bandura 1986; Gee 2008; Locke and Latham 1990) the authors discuss various advantages, including the motivation provided by specific, somewhat difficult, immediate goals. They also discuss how games support motivation and engagement by providing many paths to success, giving students the opportunity to choose smaller goals within the larger task.

Games are fun and collaborative, which means that more students have opportunities for success. Attitudes are also an important part of success. Students who feel good about a subject and their ability to do well in it will be motivated to learn. It is important to provide a positive learning environment where students are under minimal stress; meaning and understanding (rather than rote memorization) are emphasized; real-world concepts are related; and students work in well-organized groups.

Students have multiple opportunities to play the games in *Focused Mathematics Intervention*. Specific instructions for playing the games and managing the game portion of the instructional period are provided on pages 35–39.

In classrooms where competitive games may pose a problem, rules can always be modified so that harmony is achieved. Most of the games in this program are considered learning games and are not designed to be competitive in nature. However, fair and friendly competition can generate excitement, determination, motivation, independence, and challenge.

Research to Practice

Using Games to Motivate Struggling Math Learners

Each kit in *Focused Mathematics Intervention* includes six math fluency games: three Math Fluency Game Sets and three Digital Math Fluency Games. Each game provides:

- reinforcement of mathematics skills in a game format
- engaging and age-appropriate art and themes
- opportunities for individual and group play
- immediate feedback through sound effects (digital games)

Assessment

"Monitoring and record keeping provide the critical information needed to make decisions about the student's future instruction" (National Center for Learning Disabilities 2011, 5). The ability to properly diagnose and monitor students' mathematics understanding and misconceptions is imperative in mathematics intervention programs. Teachers must be able to provide instruction that is tailored to the needs of each student. In the article "RtI in Math Class," Gresham and Little (2012) recommend that educators gather assessment data from a variety of sources, including formative assessment, norm-referenced, and observations. This data should be gathered in a timely way to enable the teacher to problem solve and be proactive in instructional planning.

Formative assessments may be used by teachers to determine the point-in-time status of students' understandings and make decisions about next instructional steps (Wiliam 2010). Noted math expert Marilyn Burns also shares that formative assessment gives information to teachers about what students understand and shows possible misconceptions. Strategies such as utilizing open questions/tasks, as well as observing, listening, and reviewing student work should all be key components in formative assessment in the mathematics classroom, with the goal of using this information to guide instruction (Burns 2005).

In the article "The Bridge Between Today's Lesson and Tomorrow's," Carol Ann Tomlinson shares principles for using formative assessment to ensure a lesson sequence aligns to content goals and provides insight into student understanding to inform instruction (2014). These principles are applied in *Focused Mathematics Intervention*.

- Explain to students the role of formative assessment.
- Use assessments like Quick Checks to provide user-friendly, instructive feedback for students.
- Monitor progress informally and formally throughout a lesson using a variety of methods.
- Engage students in the formative assessment process (e.g., self-assessing using a rubric, examining teacher feedback, and developing personal goals for progressing along the learning continuum).
- Examine patterns in formative assessment data to inform student groupings, corrective feedback, and follow-up instruction.
- Use Progress Monitoring data to plan Differentiated Instruction to meet students' needs.
- Make the formative assessment process a habitual and systematically integrated part of teaching for learning.

(Tomlinson 2014)

Summative assessments can be administered in a variety of forms at the end of a lesson to measure students' mastery of the concepts and skills. A balance should exist between formative and summative assessments. Using a sports analogy, there should be more "practices" (formative assessments) than "games" (summative assessments) (Schimmer 2014). Performance-based assessments also develop mathematical thinking and problem-solving skills.

Research to Practice

Assessment

Each level of *Focused Mathematics Intervention* provides teachers with numerous opportunities for assessment.

Formative Assessment: The Pretest provides teachers with the information necessary to develop a customized program of instruction for students. This assessment can guide and inform future instructional goals. Teachers can use the Pretest to determine which lessons to teach based upon the students' skill levels.

Progress Monitoring: Each lesson in the *Student Guided Practice Book* includes a Quick Check and Math in the Real World concept task that can be used for ongoing progress monitoring.

Informal assessment opportunities are embedded throughout the lessons and activity pages to identify optimal times for teachers to observe students' conceptual understanding and skills. These data can guide future instructional decisions. Moreover, pacing plans are included to help teachers implement the program over the course of several weeks or an entire school year.

Summative Assessment: The Posttest assessment, as well as the Performance Tasks, can measure students' progress once all the selected lessons have been completed. This test provides students with the opportunity to demonstrate their mastery of the concepts taught and helps teachers re-evaluate earlier strategies or steps that will influence what follows on a student's academic or instructional path.

Components of Effective Mathematics Intervention Programs

Gersten et al. identify eight evidence-based recommendations designed specifically to reduce the number of students who struggle in mathematics (2009). These recommendations are the core foundation of an effective intervention program regardless of the instructional setting.

Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.

It is crucial to identify students who need intervention support. Screening may be in the form of interviews with students, teacher-developed screening tools, or standardized assessment tools. These diagnostic assessments not only identify students who need support, but should also identify the area(s) of weakness that need to be addressed, including misconceptions and procedural incompetencies. The *Assessment Guide* in *Focused Mathematics Intervention* offers a program-based Pretest, which helps to identify specific mathematics concepts that students need extra support to master.

Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5, and on rational numbers in grades 4 through 8.

Researchers agree that students with difficulties in mathematics often lack proficiency in computational skills (Bryant et al. 2008; Gersten, Jordan, and Flojo 2005; Calhoon et al. 2007). Quality computation instruction is important to ensure early mastery of these foundational skills (Miller et al. 2011). While *Focused Mathematics Intervention* includes lessons on each of the nationally emphasized mathematical domains, the majority of lessons focus on number sense and building conceptual and procedural knowledge.

Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.

The gradual release of responsibility model (I Do, We Do, You Do) in *Focused Mathematics Intervention* begins with the teacher modeling concepts and asking intentional questions to encourage students to share their thinking. This provides information about where students are and what support they will need. Each lesson component makes explicit connections among math ideas and representations.

Interventions should include instruction on solving word problems that is based on common underlying structures.

Problem solving is woven through each lesson in *Focused Mathematics Intervention.* A blend of conceptual and procedural understanding is developed, connecting the meaning of operations with procedural skills.

Intervention materials should include opportunities for students to work with visual representations of mathematical ideas, and interventionists should be proficient in the use of visual representations of mathematical ideas.

We know that students need experiences with concrete representations to make the connections to pictorial and numerical representations (Vygotsky 1978). *Focused Mathematics Intervention* helps students move from concrete to pictorial to numerical/abstract representations by carefully scaffolding instruction for each student. Also, the detailed lesson plans develop teachers' understanding of visual models, supporting their instructional use.

Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.

Basic facts are naturally incorporated in *Focused Mathematics Intervention*, and the math fluency games specifically address the reinforcement of basic facts and computational fluency.

Monitor the progress of students receiving supplemental instruction and other students who are at risk.

We know that intervention support without ongoing monitoring of progress does not give the information to know if students are progressing. Without this information, teachers will not know how to structure ongoing support. Assessing student progress through formative assessments (such as the Quick Checks) not only guides the intervention teacher, but also helps the classroom teacher in developing ongoing support for students in the regular mathematics class.

Include motivational strategies in Tier 2 and Tier 3 interventions.

Students who struggle often tend to become discouraged and give up. Giving these students more of the same, often drill sheets, certainly does not meet their needs for achieving success in mathematics. The activities in *Focused Mathematics Intervention* provide students with multiple experiences and opportunities to develop understanding and practice skills. Lessons are designed to scaffold student understanding and support student success. The games included in each kit are motivational and fun to play while providing ongoing practice. In addition, teachers may provide other measures of reward for student accomplishments.

Differentiation

Differentiating by Specific Needs

Today's classrooms are filled with students of varying backgrounds, levels of English proficiency, and learning styles. A teacher's ability to differentiate instruction and respond effectively to students' needs is critical to a program's success (Henry and Pianta 2011). Two factors influence a teacher's ability to use a program: instructional options that meet the needs of diverse students and having the confidence and skill to modify instruction based on those needs. *Focused Mathematics Intervention* takes these factors into account. Each lesson plan includes a variety of instructional strategies to reach students who are not yet achieving their potential, those who are performing on or above level, and English language learners (ELLs).

Below-Grade-Level Students

Below-grade-level students need concepts to be made more concrete. They may also need extra work with manipulatives and other models, such as visual models, to support conceptual understanding. Giving students extra support in the following ways allows them to feel more secure and have greater success:

- Break mathematical procedures and processes into smaller chunks, having students practice and master one part of the procedure or process. Then, add the next step, practicing the new step with the previous steps.
- Allocate extra time for guided practice.
- Prepare easy-to-follow notes of key procedural information for students to add to during the lesson.

Research to Practice

Below-Grade-Level Students

Every lesson in *Focused Mathematics Intervention* provides rich support for below-grade-level students by reteaching key concepts and skills. The activities are designed to encourage active involvement in developing mathematical thinking and provide repeated practice without losing engagement. Each lesson has differentiation built in, through engaging games, focused skill instruction, and activities for multiple learning modalities. Students are supported through teacher modeling, partner and group work, sentence frames, and concrete materials to scaffold mathematical thinking and problem solving.

English Language Learners

Mathematics intervention programs must support the content language development of English language learners. Meeting these needs can be more complex than meeting the needs of native-language struggling learners. Intervention for ELLs should engage students in meaningful activities, as well as cognitively demanding content, while scaffolding the content to ensure that students will learn successfully (Díaz-Rico and Weed 2002). Scaffolding in lessons, modeling effective strategies for learners to use, and rich vocabulary development instruction are vital for English language learners. It is not enough to simply expose English language learners to language-rich classrooms; they need specific instruction in academic vocabulary and grammar structures, intentionally provided across the content areas in order to attain mastery of content-area standards (Feldman and Kinsella 2005).

In addition to direct, explicit instruction, interactive teaching that uses techniques such as modeling and guided practice helps students master requisite skills more effectively (Goldenberg 2010).

- Create anchor charts with examples of real-world problems and steps for mathematical procedures. Incorporate graphics and realia.
- Scaffold language demands of word problems. For example, read the problem to students; simplify complex sentences and add bullets to these sentences; or add additional context to the problem to build background.
- For written responses, provide students with a sentence starter or sentence frame.

Research to Practice

English Language Learners

The instruction in academic language related to lessons in *Focused Mathematics Intervention*, and elaborations on vocabulary throughout each lesson, are particularly suited to English language learners. Additionally, the open-ended questions and sentence frames provide opportunities for students from varying backgrounds to relate to one another.

Extend Learning

According to Dorn and Soffos, the goal of teachers should be to create self-regulated students who can direct their own learning for a variety of tasks and purposes (2005). Students performing at or above grade level have the metacognitive ability to apply new concepts and vocabulary to independent work quickly and effectively; however, they sometimes face the risk of boredom. In *Can You Hear Me Now? Applying Brain Research and Technology to Engage Today's Students*, Michel and Nimz explain that "when educators promote...effort over ability, both explicitly and implicitly, all students do better" (2012, 61). To foster students' talents, the authors suggest opportunities for meaningful practice within the learners' control, setting high expectations, and providing specific encouragement for effort.

- After solving a word problem, have students create a follow-up problem or question to be explored.
- Compact the Independent Practice activity by assigning only tasks that are challenging to students, and then have students complete Extend Learning tasks.

Include motivational strategies in Tier 2 and Tier 3 interventions.

Students who struggle often tend to become discouraged and give up. Giving these students more of the same, often drill sheets, certainly does not meet their needs for achieving success in mathematics. The activities in *Focused Mathematics Intervention* provide students with multiple experiences and opportunities to develop understanding and practice skills. Lessons are designed to scaffold student understanding and support student success. The games included in each kit are motivational and fun to play while providing ongoing practice. In addition, teachers may provide other measures of reward for student accomplishments.

Research to Practice

Extend Learning

The lessons in *Focused Mathematics Intervention* provide rich mathematical opportunities for students to take math concepts and apply them successfully through the use of strategies and construction of mathematical models. The various activities throughout the lessons allow students to apply their knowledge of key concepts, necessary for content development. Teachers may use the additional activities in the Extend Learning section to challenge students while reinforcing skills taught in the lesson. All Extend Learning activity sheets can be found on the Digital Resources USB Device.

Developing Academic Vocabulary

Academic Vocabulary

In mathematics, vocabulary is highly specialized. These words are often not encountered in everyday life. Therefore, all students need an explicit introduction and explanation of these vocabulary words in order to apply them to their understanding of mathematical concepts. The task is even more difficult for English language learners. Mathematics vocabulary words are not typically words that ELLs will learn with peers. Therefore, it is up to the mathematics teacher to make certain that ELLs learn the necessary vocabulary to achieve comprehension of key mathematical concepts.

Research has consistently found a deep connection between vocabulary knowledge, reading comprehension, and academic success (Baumann, Kame'enui, and Ash 2003). Kamil and Hiebert describe vocabulary as a bridge between the "word- level processes of phonics and the cognitive processes of comprehension" (2005, 4). This is a useful way to visualize the importance of vocabulary for students who struggle with mathematics.

Mathematical language can also hinder student learning, causing students with math difficulties to focus on terms and definitions instead of the mathematical relationships involved. By using correct terminology within the context, children can integrate the words more naturally into their vocabulary (Fosnot and Hudson 2010). Students who are struggling with mathematical concepts or students who have not shown mastery of the vocabulary also need structured lessons to focus attention on the content words.

Teachers should follow these guidelines before beginning to teach the vocabulary activities in this resource.

- Frontload the lesson with vocabulary words before the students need to apply them during practice activities and problems.
- Revisit past vocabulary words in addition to current words, if a lesson requires them.
- Repeat the activity with either the same words or new words if students need more practice to correctly perform the activity.

Research to Practice

Academic Vocabulary

Each level of *Focused Mathematics Intervention* develops academic vocabulary and:

- includes an introduction of new vocabulary prior to the lesson to build conceptual understanding
- provides focused instruction on key mathematical content
- supplies a teacher glossary as a reference for key academic vocabulary
- includes a student glossary in the *Student Guided Practice Book*, providing student-friendly definitions of academic vocabulary

Developing Math Skills Using Concrete Models

Research repeatedly shows that students gain greater conceptual understanding and are more successful in demonstrating mastery of concepts when they have had a chance to concretely experience mathematical concepts using manipulatives. In addition, when students use the manipulatives, they perform better academically and have more positive attitudes toward mathematics (Leinenbach and Raymond 1996). However, many teachers, especially middle school and high school teachers, shy away from using manipulatives.

Manipulatives are usually colorful, intriguing materials constructed to illustrate and model mathematical ideas and relationships for students in all grades (Burns and Silbey 2000). Manipulatives are sometimes called math tools, or objects to think with (Kennedy, Tipps, and Johnson 2008). There are many examples of common manipulatives:

- rulers
- coins
- pattern blocks
- counters
- calculators
- base ten blocks
- connecting cubes
- algebra tiles

Children seem to learn best when they mathematize, or understand key math concepts and processes through models. To mathematize, children must learn to see, organize, and interpret through tools or models. The use of the mathematical models becomes a very important and powerful tool for reasoning (Fosnot and Hudson 2010, 21). Learning begins with a concrete representation of a mathematical concept (Cathcart et al. 2000). Manipulatives are an effective tool for students to use to build concrete representations. First, manipulatives provide an alternate route to access and develop understanding of mathematics. Second, manipulatives are intuitive for students. Problems are easier to solve when students can draw upon their practical, real-world knowledge (McNeil and Jarvin 2007).

Abstract ideas that are presented in mathematics classrooms are confusing to many students. Manipulatives support learning by creating physical models that become mental models for concepts and processes (Kennedy, Tipps, and Johnson 2008). Manipulatives help students develop the ability and confidence to see relationships and connections among the domains of mathematics: counting and cardinality, number and operations, base ten, algebraic thinking, measurement and data, geometry, and statistics and probability.

Using manipulatives effectively requires planning and organization on the part of the teacher. First and foremost, teachers must be clear about expectations while using manipulatives. They should discuss the similarities and differences between using manipulatives and playing with toys and games at home. For example, children make up their own rules when playing with toys; but at school, the teacher explains what tasks the students should complete during the class period.

Manipulatives are intended to be a tool for constructing understanding, not a means for output. The information below highlights the progression teachers need to follow when using manipulatives in order to correctly transition from concrete to abstract.

1. Explain the role of manipulatives, how they connect to an overall mathematical concept, and the expectations for student use.
2. Give students practice in using the manipulatives to explore the mathematical concept.
3. Model the mathematical concept with pictures that replace the manipulatives. Make connections between the manipulatives and the pictures.
4. Give students practice in using pictures (as a substitution for the manipulatives) to explore the mathematical concept.
5. Teach the abstract qualities of the mathematical concept. Make connections between the pictures and the equations or formulas.
6. Provide ample opportunities to practice problem-solving procedures without pictures or manipulatives.
7. Return to manipulative use when needed, repeating this entire process to move students to abstract thinking and problem solving.

Research to Practice

Manipulatives

In *Focused Mathematics Intervention*, there is one main manipulative punchout provided that can be used with the majority of the lessons, allowing for ease of preparation and management. The value of the manipulative is not in the cost of the item but in its use. Matching the manipulative to the concept is the most important strategy that any teacher can use. In this program, manipulatives are often used to help make abstract ideas more concrete.

Developing Mathematical Problem-Solving Skills

Why We Teach Problem Solving

Not many people do mathematical computation just for fun. We learn and apply mathematics to serve us in solving real-world problems in a variety of contexts. Facing a problem in real life, outside of mathematics class, calls for understanding the context, using what we know about the meaning of mathematical operations to apply a specific procedure, and knowing when an answer is reasonable. Problem solving is an area in which intervention students often struggle. Some have difficulty reading and associating meaning to worded situations. Others do not have a deep grasp of the multiple situations in which an operation can be used. Most lack a repertoire of strategies to allow them entry to a solution path. This is why we often hear students lament, "I don't get what they want us to do."

Providing students with experiences that introduce a concept in a contextual setting moves the lesson away from a focus on arithmetic skill and toward thinking about the meaning of an operation and when to use it. A link between conceptual and procedural understanding begins to take place. For example, students not only know how to subtract but also know of many situations in which subtraction is the correct operation to use.

Making Connections

The deep understanding that is based on conceptual experiences involves more than applying isolated facts or procedures. Research confirms that we learn mathematics by making connections between what we understand and new ideas. Problem solving is the context in which students can extend current understandings to new situations and make connections between procedural and conceptual understanding. Students who struggle with mathematics need more explicit instruction in how to make those connections in problem situations. For example, when students understand the connections between multiplication and division of whole numbers, they can think about division in terms of a missing factor and use what they know about multiplication to help solve division problems. Look at the following problem.

Maria reads 35 pages of her book each evening. It takes her 7 nights to complete the book. How many pages are in her book?

- In this example we know how many nights (groups) and how many pages she reads each night (number of items in a group).
- We can use multiplication to determine the number of pages.

$$35 \times 7 = 245$$

Maria's book has 245 pages.

Now look at this problem.

The book our class is reading has 245 pages. If I read 35 pages a night, how long will it take to finish reading the book?

- When a student can connect information from the first problem to this problem, the relationship between division and multiplication and the solution process becomes clear.
- In this situation, we know the total number of pages and how many pages per night, and we can determine the solution by dividing to find the number of nights.

$$245 \div 35 = 7$$

It will take 7 nights to finish the book.

In addition to the connections among mathematical ideas, students need opportunities to solve mathematics problems in everyday life, and to engage with topics that are of interest to them.

> **Research to Practice**
>
> **Making Connections**
>
> *Focused Mathematics Intervention* offers students a variety of opportunities to solve problems through the systematic, explicit, and intentional design of the lessons and activities. Students are provided opportunities to make connections and build mathematical proficiencies through real-world scenarios and problem types.

A Problem-Solving Framework

All students, especially those who need extra support, benefit from learning and applying a protocol for using problem-solving strategies. The work of George Pólya [1945] (2004) helps to provide a framework for students as they tackle a problem. Intervention students need explicit instruction and practice using the framework. The steps include:

1. **Understand the problem**. Having students read the problem either silently or aloud (or if necessary, having the teacher read the problem to the students) is the first step to understanding the problem. For many students, putting the problem in their own words helps them to make sense of the information and the question. Asking the following questions helps students who need extra support to focus on the critical parts of a problem (Gojak 2011):
    - What do you know? *Go over the information in the problem.*
    - What do you want to find out? *Focus on the question.*
    - What information in the problem will help you answer the question?
    - What information is extra? *This is the information that is not needed to answer the question.*
    - Do you need any other information to find a solution? *This question has the potential to help students identify necessary steps in multistep problems.*
    - What might be a reasonable answer to this problem? *The point here is not to answer the question but to lead students to make sense of the problem situation and the solution.*

2. **Devise a plan**. Once a student understands the problem context, he or she can begin to associate the information given and the question in terms of a mathematical idea or operation. In some problems, the plan may be directly related to the meaning of an operation. In others, specific problem-solving strategies will be helpful in planning a path to the solution. All students should have explicit instruction and multiple opportunities to use the following strategies. Intervention students need additional scaffolding in using the strategies, especially when more than one strategy might be needed.
3. **Carry out the plan**. Students who have difficulty solving problems often skip the first two steps and jump right to working on the problem. This is usually when they get stuck. Students must complete the first two steps before attempting to solve the problem. Once they understand the problem, select a strategy, and are clear on what the problem is asking, they are ready to begin the actual work of solving the problem. An important part of this step is for students to check their thinking as they progress. Are they headed in the right direction or down the wrong path? Is the strategy they selected working, or do they need to try something else? The role of the teacher in asking questions is a critical support in leading students to become more independent problem solvers. The expectation for students' written work should be that it is organized and clear. This not only lessens the possibility of getting lost in the solution process, but also supports communicating students' thinking and their representations of the mathematics. It is through clear communication and representations that students are more likely to make connections among mathematical ideas and real-life applications. The solution process, which is the plan (step 2) now put into action, leads to a deeper level of understanding and helps students apply that knowledge to new and different situations.
4. **Look back**. Too often students solve the problem, close their books, and are finished. In fact, some students might even do all of the work to "solve the problem" and never answer the question! Too many students think that the goal of mathematics is to get to the answer, and then the thinking stops. In looking back, students have the opportunity to think about their work and the reasonableness of their solution given the constraints of the problem. Additionally, opportunities to discuss their thinking with one another helps the mathematical ideas and relationships make sense. The role of the teacher in asking questions and leading discussions—especially for students who have struggled with the problem—is critical in this step.

Teachers should be purposeful in selecting problems for students to solve. Often, students who are identified as needing intervention are limited to routine problems that involve low cognitive demand, which simply provide practice for the mathematics they just learned. Struggling students should have opportunities to solve challenging, non-routine problems. The role of the teacher is to scaffold high cognitive-demand problems so that struggling students have entry to the problem. Linkages to more complex tasks may need to be more explicit for students who struggle. To avoid giving intervention students opportunities to solve rich problems is to shortchange their mathematical experiences.

Problem solving is a key reason for learning mathematics. It is through problem solving that we can look at a situation, analyze it, and determine possible solution paths and reasonable solutions. It is problem solving that makes mathematics meaningful in our daily lives.

Research to Practice

Problem Solving

Focused Mathematics Intervention systematically develops students' problem-solving skills and strategies with rich and complex Math in the Real World tasks. As students work the tasks, they are guided through the problem-solving process. Initially, to understand the problem, students paraphrase the task to their partner and then "unpack" the problem by recording key information. Next, students devise or "make a plan" and utilize problem-solving strategies. The teacher is provided with questions to focus, assess, and advance students' thinking during this phase of the process. To conclude, students "look back" and reflect on the problem-solving process by explaining their thinking. Embedded into these rich real-world tasks are many opportunities for teachers to make important points about the mathematics content and mathematical thinking.

Math in the Real World

Mathematical thinking is the key to mathematical literacy. "Mathematical thinking is a whole way of looking at things, of stripping them down to their numerical, structural, or logical essentials, and of analyzing the underlying patterns. Moreover, it involves adopting the identity of a mathematical thinker." (Devlin 2012) To develop these habits of mind, states have set forth specific mathematical processes and practices that students must master. Students are to build proficiency with these processes and practices as they master the content standards for their grade level. As students develop proficiency with the process and practice standards, they will be more successful problem solvers, use mathematics effectively and efficiently in daily life, and become college and career ready (Texas Education Agency 2012).

Research to Practice

Developing Mathematical Thinking

Focused Mathematics Intervention identifies specific mathematical practice and process standards addressed in each lesson. The program holistically develops students' proficiency with these standards throughout each lesson.

- During the Math in the Real World concept task, students practice each step of the problem-solving model.
- Students model real-world contexts using multiple representations in Math in the Real World, as well as relevant Whole-Group lessons and Differentiated Instruction.
- On many activity sheets, students explain their thinking by communicating mathematical ideas, sharing their reasoning, and analyzing relationships between concepts.
- In the lessons, students strategically select and use appropriate tools including real objects, manipulatives, paper and pencil, and technology.
- Students develop mathematical language during the Language and Vocabulary component of each lesson and practice using the language to justify mathematical ideas.
- Throughout the lessons, students reason abstractly and quantitatively by decontextualizing given information and representing it using equations, graphs, diagrams, pictures, symbols, and language as appropriate.
- Students look for and use structure by reasoning with the mathematics content in the Warm-Up, Whole-Group lesson, and Differentiated Instruction. Through step-by-step instruction, teachers guide students to analyze similarities, relationships, or patterns.
- The gradual release of responsibility lesson structure provides students with repeated experiences and opportunities to analyze relationships and patterns in the mathematics content, allowing generalizations of methods or shortcuts.
- During the We Do, You Do, and Differentiated Instruction phases, students explain through oral and written discourse the relationships and patterns found in the mathematics content, providing compelling arguments and reasoning for the problem-solving methods and shortcuts they have derived.

Developing Math Fluency Skills

There is an emphasis in national mathematics standards for students to be able to solve math problems accurately and efficiently. While fluency with key mathematics skills such as recall of basic facts is certainly expected, it is important to realize conceptual understanding is the basis for developing fluency and automaticity, especially with students who struggle and cannot depend on rote memorization. When a student understands combinations of tens, developed through many experiences using a ten frame, he or she can extend that understanding to composing and decomposing numbers to learn difficult addition facts. For example, the student can think about 9 + 6 as taking 1 from the 6 and adding it to the 9 so the fact now becomes 10 + 5, which equals 15.

Students who struggle with mathematics need many opportunities and models to build this foundational understanding before they can simply memorize their facts. It cannot be overemphasized that intervention students need more experiences than what is provided in a usual mathematics class in order to develop the conceptual understanding needed to reach a level of fluency. Developing fluency begins with conceptual understanding, strategy development through the use of appropriate models and tools, and explicitly helping students make connections between those models and basic facts. The earlier such interventions take place, the greater chance for success in not only helping students become fluent with facts, but extending the foundational understanding to more complex whole number operation concepts.

Research to Practice

Developing Math Fluency Skills

Focused Mathematics Intervention is a balanced approach designed to develop both conceptual understanding and fluency with procedures and basic facts.

- In each lesson's Warm-Up, students activate prior conceptual knowledge, review prerequisite skills, and reinforce numeracy skills.
- In the Whole-Group Lesson and Differentiated Instruction, students construct understanding of mathematical concepts and then apply that understanding to building fluency and automaticity with mathematical procedures.
- Students further practice and reinforce key fluency skills through engaging math fluency games. There are three print games and three digital games provided in each level of *Focused Mathematics Intervention*.

Pacing Plans

When planning the pacing of a curriculum program, analyze student data to determine standards on which to focus. Once a pacing plan is selected or created based on known needs of the students and/or the results of the Pretest, teachers can focus on the lessons that correlate with the items for which students did not demonstrate mastery. The Pretest is designed to determine which concepts students have already mastered and which concepts need to be mastered. Teachers can use this information to choose which lessons to cover and which lessons to skip. Even after making these data-driven decisions, teachers may still have to accelerate or decelerate the curriculum in order to meet the needs of the students in their classes. The following are a few easy ways to change the pace of the curriculum within a whole-class setting.

Ways to Accelerate the Curriculum:

- Certain mathematical concepts may come more easily to some students. If this is the case, allow less time for the practice and application of those skills and move on to the next lesson in the program.
- Skip those lessons or concepts for which students have demonstrated mastery on the Pretest.
- Reduce the number of activities that students complete in the *Student Guided Practice Book.*

Ways to Decelerate the Curriculum:

- If the concepts in a particular lesson are very challenging to the students, allow more time for each component of the lesson: modeling, guided practice, independent practice, and application games and activities.
- Use more pair or group activities to allow students to learn from one another while reinforcing their understanding of the concepts.
- Review Quick Check pages with students and have them resolve incorrect items.

The following pacing plans show three options for using this complete kit. Teachers should customize these pacing plans according to their students' needs.

Option	Instructional Time	Frequency	Material	Notes
Option 1	6 weeks (2 hours/day)	Daily	30 lessons	All lessons covered
Option 2	4 weeks (2 hours/day)	Daily	20 lessons	20 key lessons covered
Option 3	24 weeks (60 min./day)	Twice a week	24 lessons	24 key lessons covered

Note: To further adapt the program to instructional time frames, it is highly recommended that teachers give the Pretest (*Assessment Guide,* pages 19–30) to determine which standards students have not mastered. Teachers can then use the Pretest Item Analysis to analyze their students' results and select lessons to target.

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How Many Ways Can We Represent This Number? Stretch

Standards

- ✎ Understands equivalent forms of basic percents, fractions, and decimals (e.g., $\frac{1}{2}$ is equivalent to 50% is equivalent to .5) and when one form of a number might be more useful than another
- ✎ Uses models (e.g., number lines or two-dimensional and three-dimensional regions) to identify, order, and compare numbers

Overview

With the How Many Ways Can We Represent This Number? stretch, students use numbers, words, or drawings to represent a number displayed on the interactive whiteboard. The number can vary depending on the current academic focus and level of mastery of the class.

Materials

- ✎ How Many Ways Can We Represent This Number? chart
- ✎ markers

Warming Up for the Stretch

The How Many Ways Can We Represent This Number? stretch should be introduced during calendar board instruction. Model a variety of ways a number can be represented and challenge students to suggest additional representations. Demonstrate the concept of equal representations with a number that can be converted easily into several forms of representation. Students need to understand that the representation of a number may vary, but the value of each representation is the same.

How Many Ways Can We Represent This Number? Stretch *(cont.)*

Stretch Procedure

1. Display the How Many Ways Can We Represent This Number? chart on the board or the interactive whiteboard for students to see as they enter the classroom.

2. Have students write a number, number expression, number sentence, or create a picture that is an equal representation of the given number, along with their initials.

3. When all students have added a representation, call the students together for a Math Huddle. Ask students to check the accuracy of the representations and, using math language, to explain how they determined what to add to the chart. Ask students to also justify their representations.

Suggested Questions for Informal Assessment: Math Huddle

Level of Teacher Support

- Look carefully at the chart we have created. Are there any representations that you wonder about? Why do you wonder about that representation?
- Why do you think it is important to be able to represent numbers in different ways? When do we usually use words to represent numbers? When do we use numerals? When do we use pictures or diagrams? When do we use number expressions or number sentences? Why do we sometimes choose one method of representation over another?
- What math reasoning did you use to determine that your representation is equal to the number on the chart? Does anyone agree or disagree with that reasoning? Why or why not?
- How does understanding whole numbers, fractions, decimals, and percentages help you represent numbers in multiple ways? How does knowing about addition, subtraction, multiplication, and division help?
- When would we want to find other representations of a number? How would this help you if you are shopping and see something on sale? How about when you are sharing a pizza? When is it important in our daily lives to find more than one way to represent a number or value?

How Many Ways Can We Represent This Number? Stretch *(cont.)*

What It Looks Like: Stretch Snapshot

This Stretch Snapshot allows the teacher to assess students' ability to create numeric and pictorial representations of number values. As students display their representations, they are making connections between different kinds of numbers, in this case whole numbers, fractions, decimals, and percentages. It is important for the teacher to engage them in conversation about their mathematical reasoning.

This fifth grade class has completed a unit on percentages. The teacher is reinforcing students' conceptual understanding of equal representations of a percent by having them make connections between a percent, a decimal, a fraction, a diagram, and word equivalents. In this stretch, the given value is 20 percent.

The teacher notices that Susan has drawn a fraction bar divided into six parts with one part shaded. While working on fractions, Susan struggled with the conceptual understanding of what fractions represent. The teacher believes that when Susan was simplifying twenty-hundredths, she converted the percent to a fraction and simplified it to $\frac{1}{5}$ accurately, but then incorrectly drew her fraction bar using six as the base. The teacher wonders if Susan understood that the numerator indicates the number of sections to be shaded, but mistakenly thought that the denominator indicates the number of sections that should be unshaded in the fraction bar.

Teacher: *I see we have a fraction bar representation. Class, if you agree that this is an equal representation of 20 percent, thumbs up. If not, thumbs down.* (Teacher waits for response.) *Some of us don't think this represents 20 percent. Susan, I see your initials by that representation. Into how many parts is the bar divided and how many of those parts are shaded?*

Susan: *I divided it into six parts and shaded one.*

Teacher: *Explain for us how you determined that it should be divided into sixths.*

Susan: *I knew that 20 percent was the fraction 20 over 100. Then I divided it by 4 since 4 goes into 20 evenly.*

The teacher writes the fraction $\frac{20}{100}$ and places a division sign and the number 4 to the right of both the numerator and the denominator.

Teacher: *Okay, class let's divide together. 20 divided by 4 is what?*

Steve: *Five!*

How Many Ways Can We Represent This Number? Stretch *(cont.)*

What It Looks Like: Stretch Snapshot *(cont.)*

Teacher: *Excellent! Now, when we divide 100 by 4, what do we get?*

Steve: *That's 25! That was my number, five twenty-fifths.*

Teacher: *Oh, I see your answer here. Can we simplify that any further?*

Susan: *Yes, we can divide it by five over five. That would be one-fifth. So, my fraction bar is right! One part is shaded, and five parts are not.*

Teacher: *Does everyone agree with Susan?*

Steve: *I don't agree.*

Teacher: *Why not?*

Steve: *The denominator tells us how many equal parts a whole is divided into. So, the whole bar should be divided into five parts, not six.*

Susan: *Oh, I forgot. I need to fix my diagram.*

Teacher: *Very good, Susan. Now look at your diagram. Into how many equal parts should your bar be divided?*

Susan: *Five. I'm going to take one part off.*

Teacher: *You have shaded one part. How many parts should be unshaded?*

Susan: *Four parts should be unshaded, but I shaded the right amount.*

Teacher: *Very good! Come up and make that change. Which part of the fraction tells us into how many equal parts to divide the bar?*

Steve: *The denominator.*

Teacher: *What does the numerator represent?*

Susan: *The number of parts that I shade.*

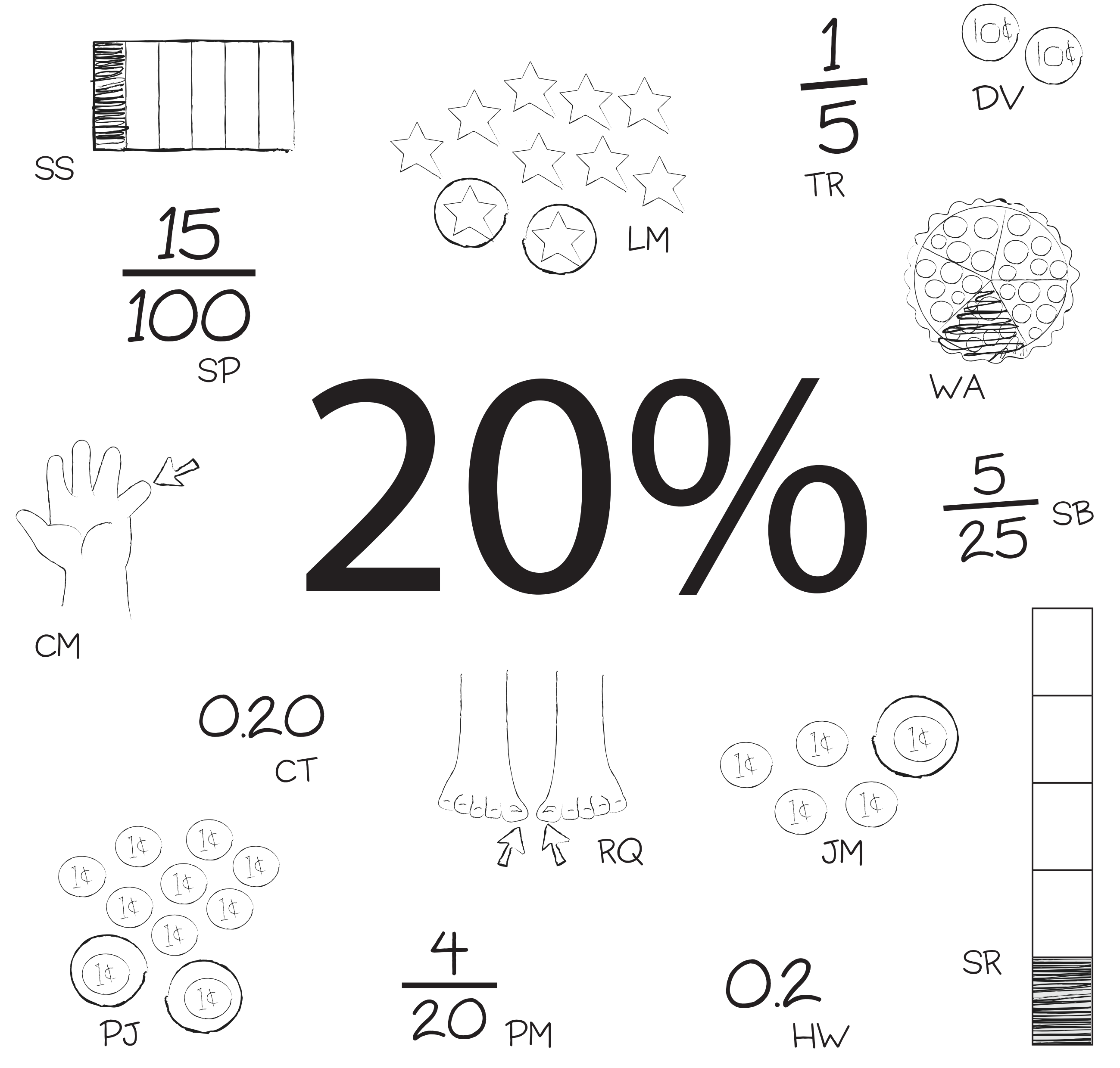
The teacher was able to diagnose Susan's misconception and to correct it. The discussion provided a review of fractions and simplifying fractions for the entire class, reinforcing student understanding of a skill previously taught.

How Many Ways Can We Represent This Number? Stretch *(cont.)*

Sample Chart

How many ways can we represent this number?

Use words, numbers, or pictures to represent this number. Add your initials to your representation.



What's My Neighbor? Stretch

Standards

- ✎ Understands the basic meaning of place value
- ✎ Understands the relative magnitude and relationships among whole numbers, fractions, decimals, and mixed numbers
- ✎ Uses explanations of the methods and reasoning behind the problem solution to determine reasonableness of and to verify results with respect to the original problem

Overview

Students demonstrate their understanding of number sequence and place value as they fill in the "neighbors" of given numbers on a hundredths chart. This stretch should not be assigned until after students have become familiar with fractions and decimals to the hundredths place as represented on a hundredths chart.

Materials

- ✎ What's My Neighbor? chart
- ✎ markers

Warming Up for the Stretch

Most students in grades three to five are familiar with a hundred chart. This stretch extends their knowledge by using a 10 by 10 grid to represent not 100, but one whole. Each cell of the grid represents $\frac{1}{100}$ or 0.01. Therefore, students should already be introduced to the concept of fractions and/or decimals to the hundredths place before using this Stretch.

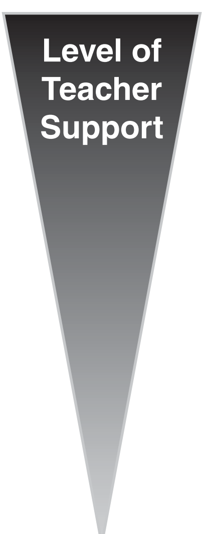
Practice the What's My Neighbor? activity using the hundredths chart during whole-class instruction. Ask the class to first identify numbers above, below, and on either side of a particular fraction or decimal on a completely filled-in hundredths chart. After several days of practice, introduce a chart with only a few cells filled in. Initially, model the process of filling in the missing numbers, thinking aloud to share the strategies used. Then, encourage students to determine the missing numbers with teacher and peer support. Once students understand the procedures and expectations, the task may become a Math Stretch to be completed independently at the beginning of the school day.

What's My Neighbor? Stretch *(cont.)*

Stretch Procedure

1. Before beginning the stretch, fill in at least one number (fraction or decimal) on the hundredths chart for every four students in the class. Leave the space above, below, immediately before, or after each filled-in number blank. Circle each of these spaces. Fill in other spaces on the chart at your discretion, based on the students' needs.
2. Display the What's My Neighbor? hundredths chart (or use the interactive whiteboard).
3. As students enter the classroom, have each of them fill in one of the circled spaces with the correct fraction or decimal, along with their initials. ***Note:*** Once students master working with hundredths, this stretch can be modified to represent other fractions or decimals. For instance, a grid with five columns and 10 rows can represent 10 wholes divided into fifths. Each row represents a whole; each cell represents one fifth. Or, the 10-by-10 grid can represent 10 wholes, each divided into tenths.
4. When every student has filled in a space, call the class together for a brief Math Huddle. Use the questions below to aid in the discussion.

Suggested Questions for Informal Assessment: Math Huddle



- ✎ Look at the fractions that have been added to our chart. Are there any that you think are incorrect? Why do you think so?
- ✎ How did you know what fraction to put in your circle? Did anyone use a different way to find the correct number?
- ✎ Do you notice any patterns when you look at the numerators above the original fractions? How do they compare to the numerators below the original fractions?
- ✎ Compare the fractions before and after the original fraction?
- ✎ Without using the hundredths chart, if you were given just one fraction, could you tell what number would be above it, below it, before it, and after it? How could you check your answers?

What's My Neighbor? Stretch *(cont.)*

What It Looks Like: Stretch Snapshot

Some students will immediately look at the tens place or the ones place in the numerator and increase or decrease the tens or ones place by 10 or by one. Others will need time to understand how place value relates to the numerator of a fraction. By allowing ample time for focused discussion guided by premeditated teacher questioning, students will develop an understanding of the relative values of fractions with like denominators. In this Stretch Snapshot, the teacher notices that although a student recorded a correct answer, this student has not applied his knowledge of place value with whole numbers to decimal fractions (i.e., fractions that convert easily into decimals, such as tenths and hundredths).

Teacher: *Dante, I see you placed $\frac{25}{100}$ in the cell below $\frac{15}{100}$. What strategy did you use to find that fraction?*

Dante: *I counted the number of empty boxes between $\frac{15}{100}$ and $\frac{25}{100}$.*

Teacher: *That was a good way to solve this problem. Class, what are some other ways you used to find the missing neighbor?*

Than: *My fraction came before $\frac{15}{100}$, so I subtracted one from the numerator. Couldn't you just add ten one-hundredths to find Dante's fraction?*

Teacher: *What do you think, class? Could we just add 10 to the numerator? Turn and talk to your elbow buddy. Tell him or her what you think and why.*

The teacher listens in to student conversations. Most students agree that this is a valid way of finding the neighbor below a given fraction. A few students still have not quite grasped this concept.

Teacher: *I have heard some good math talk. Than, will you please tell us why you would add ten one-hundredths to the fraction?*

Than: *Well, it is right underneath $\frac{15}{100}$ on the hundredths chart, so I know it will have a 5 in the ones place of numerator, and each row is 10 boxes that equals $\frac{10}{100}$. So, $\frac{15}{100}$ plus $\frac{10}{100}$ is $\frac{25}{100}$. The fraction must be $\frac{25}{100}$. It is almost like when we look at a hundredths chart except, that these are fractions.*

What's My Neighbor? Stretch *(cont.)*

What It Looks Like: Stretch Snapshot *(cont.)*

Teacher: *Why don't you change the denominator?*

Than: *That has to stay the same. All of them have 100 on the bottom.*

Teacher: *Why do they have 100 as the denominator?*

Dante: *I know! I know! The chart is divided into 100 equal parts. That's what the denominator shows.*

Teacher: *Excellent! You used what you know about place value to determine the missing fraction. Class, look at the chart and let's find other patterns that help us determine what fraction goes before, after, above, or below the given fractions. Good mathematicians also look for mathematical connections as they solve problems. Always think about how the mathematics you are working on may connect with mathematics concepts we have already learned.*

In this Math Huddle, students who are not yet seeing how place value applies to decimal fractions begin to gain an understanding of how these concepts are connected. This stretch can be extended by having students consider how to convert fractions into decimals and percentages.

What's My Neighbor? Stretch *(cont.)*

Sample Chart

What's My Neighbor?

Fill in one of the circles on this hundredths chart. Add your initials.

				$\frac{5}{100}$ TM					
			$\frac{14}{100}$ SS	$\frac{15}{100}$	$\frac{16}{100}$ DF				
		$\frac{23}{100}$ RT		$\frac{25}{100}$ DW					
	$\frac{32}{100}$ CD	$\frac{33}{100}$	$\frac{34}{100}$ LB				$\frac{38}{100}$ FW		
		$\frac{43}{100}$ VR					$\frac{48}{100}$		
	$\frac{52}{100}$ ML						$\frac{58}{100}$ CV		
$\frac{61}{100}$ TZ	$\frac{62}{100}$	$\frac{63}{100}$ JA				$\frac{67}{100}$ MK			
	$\frac{72}{100}$ SJ		$\frac{74}{100}$ PT			$\frac{77}{100}$			
		$\frac{83}{100}$ PJ	$\frac{84}{100}$	$\frac{85}{100}$ RQ		$\frac{87}{100}$ FR			
			$\frac{94}{100}$ VA						