TCM
Teacher Created Materials

Research and evidence base for:

Focused Mathematics Intervention

ESSA Evidence
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Research on Mathematics Intervention

In 21st century teaching and learning, mathematical literacy and mathematical education reform have become more important than ever. In a society that has become so technically oriented, "innumeracy" has replaced illiteracy as our principal educational gap. This is the age of science, technology, and mathematics. To have a mathematically literate society, the population needs to have an understanding of and proficiency with mathematics concepts and procedures, as well as the ability to apply that knowledge, use it to develop models, and apply those models to new situations.

The goal of mathematics education is to provide all students with the ability to use mathematics to improve their own lives, to help them become aware of their responsibilities as citizens, and to help them prepare for their futures.

In order to accomplish these goals, state departments of education, school districts, and teachers must set high expectations for all students, and mathematics education needs to be a priority at all levels. *PISA 2012 Assessment and Analytical Framework: Mathematics, Reading, Science, Problem Solving and Financial Literacy* describes the expectations students are to meet and the experiences they need to have to achieve those expectations. "This conception of mathematical literacy supports the importance of students developing a strong understanding of concepts of pure mathematics and the benefits of being engaged in explorations in the abstract world of mathematics. The construct of mathematical literacy, as defined for PISA, strongly emphasizes the need to develop students' capacity to use mathematics in context, and it is important that they have rich experiences in their mathematics classrooms to accomplish this" (PISA 2013, 25).

Mathematics education must begin at a very early age so that students develop the foundational understanding and skills necessary to achieve in mathematics. More instructional time should be dedicated to mathematics instruction, and the curriculum should focus on a depth versus breadth approach so that students have sufficient opportunities to achieve and master the content. Cameron (et al.) emphasized the importance of concepts being taught thoroughly and solidly. Students should be solving problems that require higher-level thinking and address grade-level topics (2011). Teachers need research-based curriculum solutions to provide strong instructional support that will develop mathematical proficiency among all students.

Researchers have focused their efforts in recent years on identifying essential elements of effective mathematical interventions. These include explicit, systematic, problem-based instruction in:

1. developing proficiency in number sense with whole and rational numbers
2. building accuracy and fluency in arithmetic combinations
3. building conceptual knowledge and procedural understanding
4. problem-solving

(Gersten et al. 2009)

The Need for Intervention

Students come to the classroom with different approaches to learning, various levels of mathematical proficiency, language differences, and diverse background knowledge and vocabulary understanding. Teachers must understand the development of mathematics, considering the progression of concepts, strategies, and models that can become powerful forms of representation and tools to think with (Fosnot and Hudson 2010). Student perception may also affect learning. Students approach mathematics instruction with varying levels of readiness. Some students struggle to visualize or develop understanding of abstract concepts. Other students struggle to master mathematical procedures because they do not understand the concept or the rationale for the steps of the procedure. Additionally, students may not possess strategies for attacking an unfamiliar word problem. Whatever the obstacle, it is essential that instruction is designed to meet the mathematical needs of all students before they fail. This is why intervention is critical.

There is a need on a national level for all students, especially students struggling with math, to deepen their understanding of and proficiencies in mathematical concepts and processes. In 2000, the National Council of Teachers of Mathematics (NCTM) released the "Principles and Standards for School Mathematics: An Overview," which deepened the understanding that mathematics is a combination of content and process, encouraging the expectation of standards-based teaching (NCTM 2000). Following its release, a project sponsored by the National Science Foundation and the U.S. Department of Education published "Adding It Up: Helping Children Learn Mathematics" (National Research Council 2001). This publication introduced the five strands of mathematical proficiency. The intent of the report was to ensure that students become proficient in math content and processes. This laid the groundwork for agencies such as the National Governors Association Center for Best Practices, the Council of Chief State School Officers, and state education departments to develop mathematical content and practice/process standards that focus on the conceptual and procedural understanding children must have to develop mathematical proficiency (2014). The standards are designed as progressions, each level building upon the next. The documents are interconnected works that describe the expertise that all mathematics educators should develop in their students to build their proficiencies in mathematical understanding, reasoning, and application. Ultimately, these documents, and the state standards that have evolved from them, are designed to close the opportunity gap and provide all students equal opportunity to achieve mathematical literacy.

"*Mathematical literacy* is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena" (PISA 2013, 25).

Components of Effective Mathematics Interventions

In 2008, after a review of the research in effective mathematics instruction, the National Mathematics Advisory Panel (NMAP) summarized the six main steps that need to be taken to improve math learning in U.S. schools:

1. Pre-kindergarten through grade 8 math curricula should be streamlined to emphasize a narrower set of the most critical topics in math.
2. There must be better knowledge of how students learn mathematical concepts and the benefits of intervention, conceptual understanding, fluency, automaticity, and math skill development.
3. Teachers must have strong math skills in order to teach math.
4. Math instruction should be a combination of student- and teacher-focused instruction.
5. Assessments should be strengthened to include the emphasis of the most critical math knowledge and skills.
6. More rigorous math research designed to improve best teaching practices is needed.

Research to Practice

Components of Effective Mathematics Intervention

Focused Mathematics Intervention supports the NMAP six steps to improving math learning in the following ways:

- Rigorous and explicit lesson design provides research-based practices in math instruction through the gradual release of responsibility model.
- Best practices and models for teachers are integrated through descriptions of student misconceptions and teacher background, as well as a teacher glossary.
- Formative and summative assessments help instructors diagnose, decide, and deliver rigorous instruction.

Build Mathematical Proficiencies

Students' math difficulties are often rooted in challenges with number sense, accuracy in arithmetic combinations, and problem-solving (Hanich et al. 2001). *Focused Mathematics Intervention* supports struggling math learners using systematic, explicit instruction throughout all lessons. Mathematics concepts are taught through a developmental lens, allowing strategies and mathematical models to be used as tools for instruction, student understanding, and reasoning.

Research to Practice

Build Mathematical Proficiencies

Focused Mathematics Intervention lessons provide the following:

- opportunities for students to engage in a variety of rigorous mathematical problem types
- focused instruction on key foundational skills to focus on both whole and rational numbers, arithmetic combinations, geometry, measurement, data, and probability
- active application of mathematical word/story problem skills
- multiple opportunities to explain mathematical reasoning
- reinforcement of key math fluency skills through cooperative math fluency games

Direct, Sequential, and Gradually Released Instruction

It is essential to create an engaging learning environment in which students' mathematical understanding grows through systematic, explicit modeling, with multiple opportunities for guided and independent problem solving.

A "structure for instruction that works" is the gradual release of responsibility (Fisher and Frey 2008). The teacher guides students through the following sequence to build mastery of the standard and student ownership of learning.

- **Focus Lesson (I Do)**

The teacher explicitly guides conceptual development or gives explicit instruction of the skill by activating prior knowledge, conducting think-alouds, establishing investigative questions, and modeling mathematical examples and tasks.

• **Guided Instruction (We Do)**
The teacher and students further develop understanding of the concept or build procedural proficiency by asking questions, participating in mathematical discourse, formulating/testing hypotheses, and working mathematical tasks together.

• **Collaborative (You Do)**
Students collaboratively build mastery of the concept or skill. While working through mathematical tasks, students explore mathematical conjectures, ask and respond to questions, clarify understanding, provide feedback, and reflect on their work.

• **Independent (You Do)**
Students independently solidify their mastery of the concept or skill. While working through mathematical tasks, students progress through the problem-solving process, ask themselves questions, conduct self-talk, and explain their mathematical understanding in writing.

Research to Practice

Direct, Sequential, and Gradually Released Instruction

Every *Focused Mathematics Intervention* lesson utilizes a gradual release of responsibility model (I Do, We Do, You Do). This model:

- teaches students how to be active math learners through explicit instruction of mathematics strategies
- provides students with support to ensure the successful transfer of key mathematical concepts and procedures from guided practice to independent application

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High-Yield Strategies for Increasing Student Achievement

Marzano, Pickering, and Pollock (2001) have identified nine high-yield strategies for improving instruction and student achievement: identifying similarities and differences; summarizing and note-taking; reinforcing effort and providing recognition; homework and practice; nonlinguistic representations; cooperative learning; setting objectives and providing feedback; generating and testing hypotheses and questions; and advanced organizers. These nine strategies have the greatest measurable positive effect on all student achievement, regardless of grade level or subject matter. The work of these researchers was incorporated into the development of *Focused Mathematics Intervention*.

Research to Practice

High-Yield Strategies for Increasing Student Achievement

The *Focused Mathematics Intervention* series incorporates the nine high-yield strategies in the following ways:

- Each **lesson objective is clearly set** on the lesson overview page and reinforced in the focus question(s).
- The teacher uses data from checks for understanding and formative and summative assessment to **provide feedback, reinforce effort, and provide recognition**.
- The constructivist approach of each lesson allows students to explore mathematical concepts by **asking their own questions, formulating hypotheses, and testing those hypotheses** through construction of mathematical representations, modeling, and problem-solving.
- The gradual release of responsibility lesson design includes **opportunities for practice** in each phase of instruction, as well as in Differentiated Instruction, Math Fluency Games, and Math in the Real World concept tasks.
- The Warm-Up and focus question **provide questions and cues** prior to presenting new content to activate and link to prior knowledge.
- Students may use a **math journal to take notes** during the Warm-Up and Language and Vocabulary. Students may also benefit from **writing responses, summarizing mathematical thinking, and recording model problems** during the Whole-Group lesson.
- Students participate in **cooperative learning** during the We Do, Math Fluency Games, and Math in the Real World concept tasks.
- The Warm-Up, Language and Vocabulary, and *Student Guided Practice Book* activities allow students to explore mathematical relationships and **identify similarities and differences among math concepts**.
- Through the use of math manipulatives, multiple representations, and modeling, students **utilize nonlinguistic representations** to develop conceptual understanding, transition to abstract models, and problem solve.

Using Games to Motivate Struggling Math Learners

Games are a proven source of motivation. They are an engaging way for students to develop, maintain, and reinforce mastery of essential mathematical concepts and processes. Games eliminate the tedium of most mathematics skill drills. The article "Gamification in Education: What, How, Why Bother?" by Lee and Hammer (2011) discusses the benefits and learning potential of using games in the classroom. Citing a variety of research (Locke 1991; Bandura 1986; Gee 2008; Locke and Latham 1990) the authors discuss various advantages, including the motivation provided by specific, somewhat difficult, immediate goals. They also discuss how games support motivation and engagement by providing many paths to success, giving students the opportunity to choose smaller goals within the larger task.

Games are fun and collaborative, which means that more students have opportunities for success. Attitudes are also an important part of success. Students who feel good about a subject and their ability to do well in it will be motivated to learn. It is important to provide a positive learning environment where students are under minimal stress; meaning and understanding (rather than rote memorization) are emphasized; real-world concepts are related; and students work in well-organized groups.

Students have multiple opportunities to play the games in *Focused Mathematics Intervention*. Specific instructions for playing the games and managing the game portion of the instructional period are provided on pages 35–39.

In classrooms where competitive games may pose a problem, rules can always be modified so that harmony is achieved. Most of the games in this program are considered learning games and are not designed to be competitive in nature. However, fair and friendly competition can generate excitement, determination, motivation, independence, and challenge.

Research to Practice

Using Games to Motivate Struggling Math Learners

Each kit in *Focused Mathematics Intervention* includes six math fluency games: three Math Fluency Game Sets and three Digital Math Fluency Games. Each game provides:

- reinforcement of mathematics skills in a game format
- engaging and age-appropriate art and themes
- opportunities for individual and group play
- immediate feedback through sound effects (digital games)

Assessment

"Monitoring and record keeping provide the critical information needed to make decisions about the student's future instruction" (National Center for Learning Disabilities 2011, 5). The ability to properly diagnose and monitor students' mathematics understanding and misconceptions is imperative in mathematics intervention programs. Teachers must be able to provide instruction that is tailored to the needs of each student. In the article "RtI in Math Class," Gresham and Little (2012) recommend that educators gather assessment data from a variety of sources, including formative assessment, norm-referenced assessment, and observations. This data should be gathered in a timely way to enable the teacher to problem solve and be proactive in instructional planning.

Formative assessments may be used by teachers to determine the point-in-time status of students' understandings and make decisions about next instructional steps (Wiliam 2010). Noted math expert Marilyn Burns also shares that formative assessment gives information to teachers about what students understand and shows possible misconceptions. Strategies such as utilizing open questions/tasks, as well as observing, listening, and reviewing student work should all be key components in formative assessment in the mathematics classroom, with the goal of using this information to guide instruction (Burns 2005).

In the article "The Bridge Between Today's Lesson and Tomorrow's," Carol Ann Tomlinson shares principles for using formative assessment to ensure a lesson sequence aligns to content goals and provides insight into student understanding to inform instruction (2014). These principles are applied in *Focused Mathematics Intervention*.

- Explain to students the role of formative assessment.
- Use assessments like Quick Checks to provide user-friendly, instructive feedback for students.
- Monitor progress informally and formally throughout a lesson using a variety of methods.
- Engage students in the formative assessment process (e.g., self-assessing using a rubric, examining teacher feedback, and developing personal goals for progressing along the learning continuum).
- Examine patterns in formative assessment data to inform student groupings, corrective feedback, and follow-up instruction.
- Use Progress Monitoring data to plan Differentiated Instruction to meet students' needs.
- Make the formative assessment process a habitual and systematically integrated part of teaching for learning.

(Tomlinson 2014)

Summative assessments can be administered in a variety of forms at the end of a lesson to measure students' mastery of the concepts and skills. A balance should exist between formative and summative assessments. Using a sports analogy, there should be more "practices" (formative assessments) than "games" (summative assessments) (Schimmer 2014). Performance-based assessments also develop mathematical thinking and problem-solving skills.

Research to Practice

Assessment

Each level of *Focused Mathematics Intervention* provides teachers with numerous opportunities for assessment.

Formative Assessment: The Pretest provides teachers with the information necessary to develop a customized program of instruction for students. This assessment can guide and inform future instructional goals. Teachers can use the Pretest to determine which lessons to teach based upon the students' skill levels.

Progress Monitoring: Each lesson in the *Student Guided Practice Book* includes a Quick Check and Math in the Real World concept task that can be used for ongoing progress monitoring.

Informal assessment opportunities are embedded throughout the lessons and activity pages to identify optimal times for teachers to observe students' conceptual understanding and skills. These data can guide future instructional decisions. Moreover, pacing plans are included to help teachers implement the program over the course of several weeks or an entire school year.

Summative Assessment: The Posttest assessment, as well as the Performance Tasks, can measure students' progress once all the selected lessons have been completed. This test provides students with the opportunity to demonstrate their mastery of the concepts taught and helps teachers re-evaluate earlier strategies or steps that will influence what follows on a student's academic or instructional path.

Components of Effective Mathematics Intervention Programs

Gersten et al. identify eight evidence-based recommendations designed specifically to reduce the number of students who struggle in mathematics (2009). These recommendations are the core foundation of an effective intervention program regardless of the instructional setting.

1. Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at-risk.

It is crucial to identify students who need intervention support. Screening may be in the form of interviews with students, teacher-developed screening tools, or standardized assessment tools. These diagnostic assessments not only identify students who need support but should also identify the area(s) of weakness that need to be addressed, including misconceptions and procedural incompetencies. The *Assessment Guide* in *Focused Mathematics Intervention* offers a program-based Pretest, which helps identify specific mathematics concepts that students need extra support to master.

2. Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5, and on rational numbers in grades 4 through 8.

Researchers agree that students with difficulties in mathematics often lack proficiency in computational skills (Bryant et al. 2008; Gersten, Jordan, and Flojo 2005; Calhoon et al. 2007). Quality computation instruction is important to ensure early mastery of these foundational skills (Miller et al. 2011). While *Focused Mathematics Intervention* includes lessons on each of the nationally emphasized mathematical domains, the majority of lessons focus on number sense and building conceptual and procedural knowledge.

3. Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.

The gradual release of responsibility model (I Do, We Do, You Do) in *Focused Mathematics Intervention* begins with the teacher modeling concepts and asking intentional questions to encourage students to share their thinking. This provides information about where students are and what support they will need. Each lesson component makes explicit connections among math ideas and representations.

4. Interventions should include instruction on solving word problems that is based on common underlying structures.

Problem solving is woven through each lesson in *Focused Mathematics Intervention.* A blend of conceptual and procedural understanding is developed, connecting the meaning of operations with procedural skills.

5. Intervention materials should include opportunities for students to work with visual representations of mathematical ideas, and interventionists should be proficient in the use of visual representations of mathematical ideas.

We know that students need experiences with concrete representations to make the connections to pictorial and numerical representations (Vygotsky 1978). *Focused Mathematics Intervention* helps students move from concrete to pictorial to numerical/abstract representations by carefully scaffolding instruction for each student. Also, the detailed lesson plans develop teachers' understanding of visual models, supporting their instructional use.

6. Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.

Basic facts are naturally incorporated in *Focused Mathematics Intervention*, and the math fluency games specifically address the reinforcement of basic facts and computational fluency.

7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.

We know that intervention support without ongoing monitoring of progress does not give the information to know if students are progressing. Without this information, teachers will not know how to structure ongoing support. Assessing student progress through formative assessments (such as the Quick Checks) not only guides the intervention teacher, but also helps the classroom teacher in developing ongoing support for students in the regular mathematics class.

8. Include motivational strategies in Tier 2 and Tier 3 interventions.

Students who struggle often tend to become discouraged and give up. Giving these students more of the same, often drill sheets, certainly does not meet their needs for achieving success in mathematics. The activities in *Focused Mathematics Intervention* provide students with multiple experiences and opportunities to develop understanding and practice skills. Lessons are designed to scaffold student understanding and support student success. The games included in each kit are motivational and fun to play while providing ongoing practice. In addition, teachers may provide other measures of reward for student accomplishments.

Differentiation

Differentiating by Specific Needs

Today's classrooms are filled with students of varying backgrounds, levels of English proficiency, and learning styles. A teacher's ability to differentiate instruction and respond effectively to students' needs is critical to a program's success (Henry and Pianta 2011). Two factors influence a teacher's ability to use a program: instructional options that meet the needs of diverse students and having the confidence and skill to modify instruction based on those needs. *Focused Mathematics Intervention* takes these factors into account. Each lesson plan includes a variety of instructional strategies to reach students who are not yet achieving their potential, those who are performing on or above level, and English language learners (ELLs).

Below-Grade-Level Support

Students working below grade level need concepts to be made more concrete. They may also need extra work with manipulatives and other models, such as visual models, to support conceptual understanding. Giving students extra support in the following ways allows them to feel more secure and have greater success:

- Break mathematical procedures and processes into smaller chunks, having students practice and master one part of the procedure or process. Then, add the next step, practicing the new step with the previous steps.
- Allocate extra time for guided practice.
- Prepare easy-to-follow notes of key procedural information for students to add to during the lesson.

Research to Practice

Below-Grade-Level Support

Every lesson in *Focused Mathematics Intervention* provides rich support for below-grade-level students by reteaching key concepts and skills. The activities are designed to encourage active involvement in developing mathematical thinking and provide repeated practice without losing engagement. Each lesson has differentiation built in, through engaging games, focused skill instruction, and activities for multiple learning modalities. Students are supported through teacher modeling, partner and group work, sentence frames, and concrete materials to scaffold mathematical thinking and problem solving.

English Language Learners

Mathematics intervention programs must support the content language development of English language learners. Intervention for ELLs should engage students in meaningful activities, as well as cognitively demanding content, while scaffolding the content to ensure that students will learn successfully (Díaz-Rico and Weed 2002). Scaffolding in lessons, modeling effective strategies for learners to use, and rich vocabulary development instruction are vital for English language learners. It is not enough to simply expose English language learners to language-rich classrooms; they need specific instruction in academic vocabulary and grammar structures, intentionally provided across the content areas in order to attain mastery of content-area standards
(Feldman and Kinsella 2005).

In addition to direct, explicit instruction, interactive teaching that uses techniques such as modeling and guided practice helps students master requisite skills more effectively (Goldenberg 2010).

- Create anchor charts with examples of real-world problems and steps for mathematical procedures. Incorporate graphics and realia.
- Scaffold language demands of word problems. For example, read the problem to students; simplify complex sentences and add bullets to these sentences; or add additional context to the problem to build background.
- For written responses, provide students with a sentence starter or sentence frame.

Research to Practice

English Language Learners

The instruction in academic language related to lessons in *Focused Mathematics Intervention*, and elaborations on vocabulary throughout each lesson, are particularly suited to English language learners. Additionally, the open-ended questions and sentence frames provide opportunities for students from varying backgrounds to relate to one another.

Extend Learning

According to Dorn and Soffos, the goal of teachers should be to create self-regulated students who can direct their own learning for a variety of tasks and purposes (2005). Students performing at or above grade level have the metacognitive ability to apply new concepts and vocabulary to independent work quickly and effectively; however, they sometimes face the risk of boredom. In *Can You Hear Me Now? Applying Brain Research and Technology to Engage Today's Students*, Michel and Nimz explain that "when educators promote...effort over ability, both explicitly and implicitly, all students do better" (2012, 61). To foster students' talents, the authors suggest opportunities for meaningful practice within the learners' control, setting high expectations, and providing specific encouragement for effort.

- After solving a word problem, have students create a follow-up problem or question to be explored.
- Accelerate the Independent Practice activity by assigning only tasks that are challenging to students, and then have students complete Extend Learning tasks.

Research to Practice

Extend Learning

The lessons in *Focused Mathematics Intervention* provide rich mathematical opportunities for students to take math concepts and apply them successfully through the use of strategies and construction of mathematical models. The various activities throughout the lessons allow students to apply their knowledge of key concepts, necessary for content development. Teachers may use the additional activities in the Extend Learning section to challenge students while reinforcing skills taught in the lesson.

Developing Mathematical Content Vocabulary

Mathematical Content Vocabulary

In mathematics, vocabulary is highly specialized. These words are often not encountered in everyday life. Therefore, all students need an explicit introduction and explanation of these vocabulary words in order to apply them to their understanding of mathematical concepts. The task is even more difficult for English language learners. Mathematics vocabulary words are not typically words that ELLs will learn with peers. Therefore, it is up to the mathematics teacher to make certain that ELLs learn the necessary vocabulary to achieve comprehension of key mathematical concepts.

Research has consistently found a deep connection between vocabulary knowledge, reading comprehension, and academic success (Baumann, Kame'enui, and Ash 2003). Kamil and Hiebert describe vocabulary as a bridge between the "word-level processes of phonics and the cognitive processes of comprehension" (2005, 4). This is a useful way to visualize the importance of vocabulary for students who struggle with mathematics.

Mathematical language can also hinder student learning, causing students with math difficulties to focus on terms and definitions instead of the mathematical relationships involved. By using correct terminology within the context, children can integrate the words more naturally into their vocabulary (Fosnot and Hudson 2010). Students who are struggling with mathematical concepts or students who have not shown mastery of the vocabulary also need structured lessons to focus attention on the content words.

Teachers should follow these guidelines before beginning to teach the vocabulary activities in this resource.

- Frontload the lesson with vocabulary words before the students need to apply them during practice activities and problems.
- Revisit past vocabulary words in addition to current words, if a lesson requires them.
- Repeat the activity with either the same words or new words if students need more practice to correctly perform the activity.

Research to Practice

Academic Vocabulary

Each level of *Focused Mathematics Intervention* develops academic vocabulary and:

- includes an introduction of new vocabulary prior to the lesson to build conceptual understanding
- provides focused instruction on key mathematical content
- supplies a teacher glossary as a reference for key academic vocabulary
- includes a student glossary in the *Student Guided Practice Book*, providing student-friendly definitions of academic vocabulary

Developing Math Skills Using Concrete Models

Research repeatedly shows that students gain greater conceptual understanding and are more successful in demonstrating mastery of concepts when they have had a chance to concretely experience mathematical concepts using manipulatives. In addition, when students use manipulatives, they perform better academically and have more positive attitudes toward mathematics (Leinenbach and Raymond 1996). However, many teachers, especially middle school and high school teachers, shy away from using manipulatives.

Manipulatives are usually colorful, intriguing materials constructed to illustrate and model mathematical ideas and relationships for students in all grades (Burns and Silbey 2000). Manipulatives are sometimes called math tools, or objects to think with (Kennedy, Tipps, and Johnson 2008). There are many examples of common manipulatives:

- rulers
- coins
- pattern blocks
- counters
- calculators
- base-ten blocks
- connecting cubes
- algebra tiles

Children seem to learn best when they *mathematize,* or understand key math concepts and processes through models. To mathematize, children must learn to see, organize, and interpret through tools or models. The use of the mathematical models becomes a very important and powerful tool for reasoning (Fosnot and Hudson 2010, 21). Learning begins with a concrete representation of a mathematical concept (Cathcart et al. 2000). Manipulatives are an effective tool for students to use to build concrete representations. First, manipulatives provide an alternate route to access and develop understanding of mathematics. Second, manipulatives are intuitive for students. Problems are easier to solve when students can draw upon their practical, real-world knowledge (McNeil and Jarvin 2007).

Abstract ideas that are presented in mathematics classrooms are confusing to many students. Manipulatives support learning by creating physical models that become mental models for concepts and processes (Kennedy, Tipps, and Johnson 2008). Manipulatives help students develop the ability and confidence to see relationships and connections among the domains of mathematics: counting and cardinality, number and operations, base ten, algebraic thinking, measurement and data, geometry, and statistics and probability.

Using manipulatives effectively requires planning and organization on the part of the teacher. First and foremost, teachers must be clear about expectations while using manipulatives. They should discuss the similarities and differences between using manipulatives and playing with toys and games at home. For example, children make up their own rules when playing with toys; but at school, the teacher explains what tasks the students should complete during the lesson.

Manipulatives are intended to be tools for constructing understanding, not a means for output. The information below highlights the progression teachers need to follow when using manipulatives to help students transition from concrete to abstract.

1. Explain the role of manipulatives, how they connect to an overall mathematical concept, and the expectations for student use.
2. Give students practice in using the manipulatives to explore the mathematical concept.
3. Model the mathematical concept with pictures that replace the manipulatives. Make connections between the manipulatives and the pictures.
4. Give students practice in using pictures (as a substitution for the manipulatives) to explore the mathematical concept.
5. Teach the abstract qualities of the mathematical concept. Make connections between the pictures and the equations or formulas.
6. Provide ample opportunities to practice problem-solving procedures without pictures or manipulatives.
7. Return to manipulative use when needed, repeating this entire process to move students to abstract thinking and problem-solving.

Research to Practice

Manipulatives

In *Focused Mathematics Intervention*, there is one main manipulative punchout provided that can be used with the majority of the lessons, allowing for ease of preparation and management. The value of the manipulative is not in the cost of the item but in its use. Matching the manipulative to the concept is the most important strategy that any teacher can use. In this program, manipulatives are often used to help make abstract ideas more concrete.

Developing Mathematical Problem-Solving Skills

Why We Teach Problem-Solving

Not many people do mathematical computation just for fun. We learn and apply mathematics to serve us in solving real-world problems in a variety of contexts. Facing a problem in real life, outside of mathematics class, calls for understanding the context, using what we know about the meaning of mathematical operations to apply a specific procedure, and knowing when an answer is reasonable. Problem-solving is an area in which intervention students often struggle. Some have difficulty reading and associating meaning to word problems. Others do not have a deep grasp of the multiple situations in which an operation can be used. Most lack a repertoire of strategies to allow them entry to a solution path. This is why we often hear students lament, "I don't get what they want us to do."

Providing students with experiences that introduce a concept in a contextual setting moves the lesson away from a focus on arithmetic skill and toward thinking about the meaning of an operation and when to use it. A link between conceptual and procedural understanding begins to take place. For example, students not only know how to subtract but also know of many situations in which subtraction is the correct operation to use.

Making Connections

The deep understanding that is based on conceptual experiences involves more than applying isolated facts or procedures. Research confirms that we learn mathematics by making connections between what we understand and new ideas. Problem-solving is the context in which students can extend current understandings to new situations and make connections between procedural and conceptual understanding. Students who struggle with mathematics need more explicit instruction in how to make those connections in problem situations. For example, when students understand the connections between multiplication and division of whole numbers, they can think about division in terms of a missing factor and use what they know about multiplication to help solve division problems. Look at the following problem.

Maria reads 35 pages of her book each evening. It takes her 7 nights to complete the book. How many pages are in her book?

- In this example we know how many nights (groups) and how many pages she reads each night (number of items in a group).
- We can use multiplication to determine the number of pages.

$$35 \times 7 = 245$$

Maria's book has 245 pages.

Now look at this problem.

The book our class is reading has 245 pages. If I read 35 pages a night, how long will it take to finish reading the book?

- When a student can connect information from the first problem to this problem, the relationship between division and multiplication and the solution process becomes clear.
- In this situation, we know the total number of pages and how many pages per night, and we can determine the solution by dividing to find the number of nights.

245 ÷ 35 = 7

It will take 7 nights to finish the book.

In addition to the connections among mathematical ideas, students need opportunities to solve mathematics problems in everyday life, and to engage with topics that are of interest to them.

Research to Practice

Making Connections

Focused Mathematics Intervention offers students a variety of opportunities to solve problems through the systematic, explicit, and intentional design of the lessons and activities. Students are provided opportunities to make connections and build mathematical proficiencies through real-world scenarios and problem types.

A Problem-Solving Framework

All students, especially those who need extra support, benefit from learning and applying a protocol for using problem-solving strategies. The work of George Pólya [1945] (2004) helps to provide a framework for students as they tackle a problem. Intervention students need explicit instruction and practice using the framework. The steps include:

1. **Understand the problem**. Having students read the problem either silently or aloud (or if necessary, having the teacher read the problem to the students) is the first step to understanding the problem. For many students, putting the problem in their own words helps them to make sense of the information and the question. Asking the following questions helps students who need extra support to focus on the critical parts of a problem (Gojak 2011):
    - What do you know? *Go over the information in the problem.*
    - What do you want to find out? *Focus on the question.*
    - What information in the problem will help you answer the question?
    - What information is extra? *This is the information that is not needed to answer the question.*
    - Do you need any other information to find a solution? *This question has the potential to help students identify necessary steps in multistep problems.*
    - What might be a reasonable answer to this problem? *The point here is not to answer the question but to lead students to make sense of the problem situation and the solution.*

2. **Devise a plan**. Once a student understands the problem context, he or she can begin to associate the information given and the question in terms of a mathematical idea or operation. In some problems, the plan may be directly related to the meaning of an operation. In others, specific problem-solving strategies will be helpful in planning a path to the solution. All students should have explicit instruction and multiple opportunities to use the following strategies. Intervention students need additional scaffolding in using the strategies, especially when more than one strategy might be needed.
3. **Carry out the plan**. Students who have difficulty solving problems often skip the first two steps and jump right to working on the problem. This is usually when they get stuck. Students must complete the first two steps before attempting to solve the problem. Once they understand the problem, select a strategy, and are clear on what the problem is asking, they are ready to begin the actual work of solving the problem. An important part of this step is for students to check their thinking as they progress. Are they headed in the right direction or down the wrong path? Is the strategy they selected working, or do they need to try something else? The role of the teacher in asking questions is a critical support in leading students to become more independent problem solvers. The expectation for students' written work should be that it is organized and clear. This not only lessens the possibility of getting lost in the solution process, but also supports communicating students' thinking and their representations of the mathematics. It is through clear communication and representations that students are more likely to make connections among mathematical ideas and real-life applications. The solution process, which is the plan (step 2) now put into action, leads to a deeper level of understanding and helps students apply that knowledge to new and different situations.
4. **Look back**. Too often students solve the problem, close their books, and are finished. In fact, some students might even do all of the work to "solve the problem" and never answer the question! Too many students think that the goal of mathematics is to get to the answer, and then the thinking stops. In looking back, students have the opportunity to think about *their* work and the reasonableness of their solution given the constraints of the problem. Additionally, opportunities to discuss their thinking with one another helps the mathematical ideas and relationships make sense. The role of the teacher in asking questions and leading discussions—especially for students who have struggled with the problem—is critical in this step.

Teachers should be purposeful in selecting problems for students to solve. Often, students who are identified as needing intervention are limited to routine problems that involve low cognitive demand, which simply provide practice for the mathematics they just learned. Struggling students should have opportunities to solve challenging, non-routine problems. The role of the teacher is to scaffold high cognitive-demand problems so that struggling students have entry to the problem. Linkages to more complex tasks may need to be more explicit for students who struggle. To avoid giving intervention students opportunities to solve rich problems is to shortchange their mathematical experiences.

Problem solving is a key reason for learning mathematics. It is through problem solving that we can look at a situation, analyze it, and determine possible solution paths and reasonable solutions. It is problem-solving that makes mathematics meaningful in our daily lives.

TCM Teacher Created Materials

Research to Practice

Problem-Solving

Focused Mathematics Intervention systematically develops students' problem-solving skills and strategies with rich and complex Math in the Real World tasks. As students work the tasks, they are guided through the problem-solving process. Initially, to understand the problem, students paraphrase the task to their partner and then "unpack" the problem by recording key information. Next, students devise or "make a plan" and utilize problem-solving strategies. The teacher is provided with questions to focus, assess, and advance students' thinking during this phase of the process. To conclude, students "look back" and reflect on the problem-solving process by explaining their thinking. Embedded into these rich real-world tasks are many opportunities for teachers to make important points about the mathematics content and mathematical thinking.

Math in the Real World

Mathematical thinking is the key to mathematical literacy. "Mathematical thinking is a whole way of looking at things, of stripping them down to their numerical, structural, or logical essentials, and of analyzing the underlying patterns. Moreover, it involves adopting the identity of a mathematical thinker." (Devlin 2012) To develop these habits of mind, states have set forth specific mathematical processes and practices that students must master. Students are to build proficiency with these processes and practices as they master the content standards for their grade level. As students develop proficiency with the process and practice standards, they will be more successful problem solvers, use mathematics effectively and efficiently in daily life, and become college and career ready (Texas Education Agency 2012).

Research to Practice

Developing Mathematical Thinking

Focused Mathematics Intervention identifies specific mathematical practice and process standards addressed in each lesson. The program holistically develops students' proficiency with these standards throughout each lesson.

- During the Math in the Real World concept task, students practice each step of the problem-solving model.
- Students model real-world contexts using multiple representations in Math in the Real World, as well as relevant Whole-Group Lessons and Differentiated Instruction.
- On many activity sheets, students explain their thinking by communicating mathematical ideas, sharing their reasoning, and analyzing relationships between concepts.
- In the lessons, students strategically select and use appropriate tools including real objects, manipulatives, paper and pencil, and technology.
- Students develop mathematical language during the Language and Vocabulary component of each lesson and practice using the language to justify mathematical ideas.
- Throughout the lessons, students reason abstractly and quantitatively by decontextualizing given information and representing it using equations, graphs, diagrams, pictures, symbols, and language as appropriate.
- Students look for and use structure by reasoning with the mathematics content in the Warm-Up, Whole-Group Lesson, and Differentiated Instruction. Through step-by-step instruction, teachers guide students to analyze similarities, relationships, or patterns.
- The gradual release of responsibility lesson structure provides students with repeated opportunities to analyze relationships and patterns in the mathematics content, allowing generalizations of methods or shortcuts.
- During the We Do, You Do, and Differentiated Instruction phases, students explain through oral and written discourse the relationships and patterns found in the mathematics content, providing compelling arguments and reasoning for the problem-solving methods and shortcuts they have derived.

Developing Math Fluency Skills

There is an emphasis in national mathematics standards for students to be able to solve math problems accurately and efficiently. While fluency with key mathematics skills such as recall of basic facts is certainly expected, it is important to realize conceptual understanding is the basis for developing fluency and automaticity, especially with students who struggle and cannot depend on rote memorization. When a student understands combinations of tens, developed through many experiences using a ten frame, they can extend that understanding to composing and decomposing numbers to learn difficult addition facts. For example, the student can think about 9 + 6 as taking 1 from the 6 and adding it to the 9 so the fact now becomes 10 + 5, which equals 15.

Students who struggle with mathematics need many opportunities and models to build this foundational understanding before they can simply memorize their facts. It cannot be overemphasized that intervention students need more experiences than what is provided in a usual mathematics class in order to develop the conceptual understanding needed to reach a level of fluency. Developing fluency begins with conceptual understanding, strategy development through the use of appropriate models and tools, and explicitly helping students make connections between those models and basic facts. The earlier such interventions take place, the greater chance for success in not only helping students become fluent with facts, but also extending their foundational understanding to more complex whole-number operation concepts.

Research to Practice

Developing Math Fluency Skills

Focused Mathematics Intervention is a balanced approach designed to develop both conceptual understanding and fluency with procedures and basic facts.

- In each lesson's Warm-Up, students activate prior conceptual knowledge, review prerequisite skills, and reinforce numeracy skills.
- In the Whole-Group Lesson and Differentiated Instruction, students construct understanding of mathematical concepts and then apply that understanding to building fluency and automaticity with mathematical procedures.
- Students further practice and reinforce key fluency skills through engaging math fluency games. There are three print games and three digital games provided in each level of *Focused Mathematics Intervention*.

Data Study

Overview

Teacher Created Materials values scientific research and evidence to support programming and ultimately increase student achievement. For the purpose of this study, Teacher Created Materials partnered with Bethune-Bowman Elementary and Middle School, a combined campus in Rowesville, South Carolina to conduct a correlational study with Focused Mathematics Intervention. The study was conducted during the Mohawk Mania Extended Year Camp for five weeks in the summer of 2015. This report presents the evaluation design and student performance results.

Study Design

The intent of this study was to assess the effectiveness of Focused Mathematics Intervention in supporting students to attain mathematical concepts and build mathematical fluency. The study used a correlational research design, examining the strength of the relationship between Focused Mathematics Intervention used for mathematics instruction and student achievement through mastery of skills and standards.

Participants and Setting

Bethune-Bowman Elementary and Middle School are part of Orangeburg Consolidated 5 School District in South Carolina. School data for 2014 a total of 169 K-8th grade students indicated that the school was average in both absolute rating and growth rating, showing adequate progress for the prior school year. (https://ed.sc.gov/assets/reportCards/2014/elem/s/e3805012.pdf)

A total of 169 students in grades K–8 participated throughout the entire intervention and study with less than a 3% attrition rate. The total school enrollment and demographic information is representative of the students who participated in the study and mimic the school neighborhood demographics.

Enrollment

- Elementary: 295
- Middle/High School: 343

School Population

- African-American
  - ✔ Elementary: 85%
  - ✔ Middle/High School: 90%
- Caucasian
  - ✔ Elementary: 11%
  - ✔ Middle/High School: 8%
- Hispanic
  - ✔ Elementary: 4%
  - ✔ Middle/High School: 2%
- 37.3% Poverty Level
- Title I District

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Implementation

The purpose of the implementation of Focused Mathematics Intervention was to reinforce mathematical concepts and assist teachers in building mathematical fluency in students. Focused Mathematics Intervention was implemented during Mohawk Mania Extended Year Camp for twenty-five days in June and July 2015. This program was open to all students within both Bethune-Bowman Elementary and Middle/High School campuses. The extended-year program consisted of 2.5 hours of instructional time per day, 4 days per week, for 25 days.

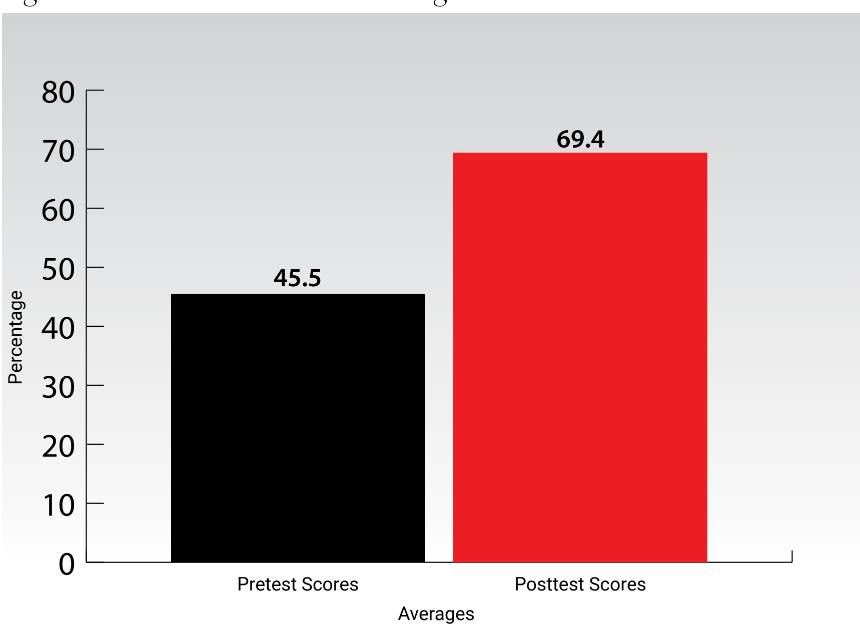
Multiple measures were used to assess student achievement, program implementation, and overall impressions of the intervention. To evaluate student progress, teachers used an aligned pretest and posttest. These assessments evaluate mathematical concepts. Each assessment examines skills and standard mastery. In order to measure teacher perspectives, a survey collected educator feedback.

Results

Upon comparing the data from Focused Mathematics Intervention pretest scores against posttest scores, evidence showed that the students who participated in the program made overall significant gains in mathematical concepts aligned to the South Carolina College and Career Readiness Mathematics Standards. Statistical analysis was conducted on all students who remained in the study.

Students showed an average gain of 23.8%. (See Figure A.)

Figure A: Pretest and Posttest Averages



The average gain of 23.8% is equivalent to six additional mathematical concepts mastered, which showcases growth of more than one skill each week of instruction. The effect size of the data is significant, indicating growth as a result of the intervention.

Surveys

Surveys collected from educators in the data study shared the following:

"The information, tools, and materials were accessible and made things a lot smoother in the classroom for our summer program. The lesson outlines and quick checks provided an awesome foundation for my students. Also, it was great to have a layout plan according to the number of weeks we would be in session. The layout gave me the opportunity to go out of sequence if needed to meet the needs of my students individually and as a class. Thank you so much for the resources!"

–1st Grade Teacher

"The goal of raising expectations and achievement gains for what students should learn and be able to do academically makes it increasingly important to use the precise instructional materials in math that are effective at improving student achievement. Teacher Created Materials' math kits are just the right fit. The curricular process begins with a shared experience, which is an activity, task, or game that is open-ended; lesson focus; language and vocabulary; challenge question; and quick checks and can be a stand-alone experience or one built on students' life experiences. As a teacher, I appreciated that I didn't have to pull materials for math intervention time blocks for my students, therefore leaving more time to plan for the mini-lessons."

–2nd Grade Teacher

"Teacher Created Materials made it easy to prepare for lessons. I really appreciated the Quick Check for checking for student understanding. The Fluency Games were engaging and fun! "

–8th Grade Teacher

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